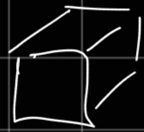
Es. 1 Scatola a forma 

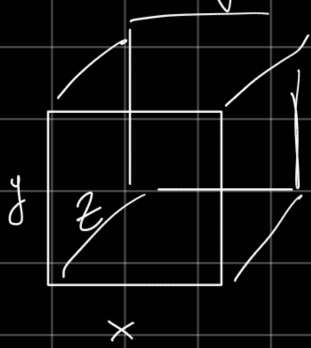
$V_0 : 1000 \, cm^3$

Trovare le dimensioni x, y, z tali

da minimizzare la superficie totale

f è la superficie.

$$f(x,y,z) = 2xy + 2zy + 2xz$$



ricaviamo una variabile dalle altre due.

$$V = xyz$$

$V_0 : 1000$

$$\bar{f}(x,y) = f(x,y,z) = 2xy + 2\frac{V_0}{x} + \frac{2V_0}{y}$$

1) trovo P. Staz.

$$f_x = 2y - \frac{2V_0}{x^2} \qquad f_y = 2x - \frac{2V_0}{y^2}$$

$$\begin{cases} f_x = 0 \\ f_y = 0 \end{cases} \qquad \begin{cases} 2y - \dfrac{2V_0}{x^2} = 0 \\ 2x - \dfrac{2V_0}{y^2} = 0 \end{cases}$$

vedo se si annullano

$$\begin{cases} y = \dfrac{V_0}{x^2} \\ 2x - \dfrac{2V_0}{\left(\dfrac{V_0}{x^2}\right)^2} = 0 \end{cases} \qquad \begin{cases} y = \dfrac{V_0}{x^2} \\ 2x - \dfrac{x^4}{V_0} = 0 \end{cases}$$

$$\begin{cases} y = \dfrac{V_0}{x^2} \\ \underbrace{2x\left(2 - \dfrac{x^3}{V_0}\right) = 0}_{\text{non ci piace... è useless}} \end{cases} \qquad \begin{cases} y = \dfrac{V_0}{x^2} \\ 2 - \dfrac{x^3}{V_0} = 0 \end{cases}$$

$$\begin{cases} y = \dfrac{V_0}{x^2} \\ 1 = \dfrac{x^3}{V_0} \end{cases} \quad x^3 = 1V_0 \quad x = \sqrt[3]{V_0} \qquad y = 10 \qquad x = 10$$

$P(10,10)$ è un punto stazionario

2) Matrice Hessiana

per vedere se è un punto di sella ecc.

$$H = \begin{pmatrix} h_{xx} & h_{xy} \\ h_{yx} & h_{yy} \end{pmatrix} = \begin{pmatrix} \dfrac{4V_0}{x^3} & 2 \\ 2 & \dfrac{4V_0}{y^3} \end{pmatrix}$$

$$h_{xx} = \frac{4V_0}{x^3} \qquad h_{yy} = \frac{4V_0}{y^3}$$

$$h_{xy} = 2$$

$Det_H = \lambda_1 \lambda_2$

$Tr_H = \lambda_1 + \lambda_2$

$Det_H > 0 \quad \lambda_1 \cdot \lambda_2 > 0 \Rightarrow \lambda_1, \lambda_2$ concordi

$tr_H > 0 \quad \lambda_1 + \lambda_2 > 0 \Rightarrow \lambda_1, \lambda_2$ positivi

$$Det_H(P_1) = \left[\frac{4V_0}{x^3} \cdot \frac{4V_0}{y^3} - 2 \cdot 2\right] =$$

$$= \frac{16V_0^2}{V_0 \cdot V_0} - 4 = 12 > 0$$

$$Tr_H(P) = 4V_0\left(\frac{1}{x^3} + \frac{1}{y^3}\right)_P = 4V_0\left(\frac{1}{V_0} + \frac{1}{V_0}\right) = 8 > 0$$

H def. positiva in P $\Rightarrow$ P è minimo

$x = 10 \quad y = 10 \quad z = 10$

Es 2. Calcolare Max e Min di $f(x,y) = xy$ ristretta alla circonferenza UNITARIA centrata in $(0,0)$

METODO 1: Moltiplicatori di Lagrange

$C = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 - 1 = 0\}$

$\mathcal{L}(x,y,\lambda) = f(x,y) + \lambda g(x,y)$

$\begin{cases} g = 0 \\ \nabla f = \lambda \nabla g \end{cases}$ ⟸ contiene le info della funz. vera e propria

$\begin{cases} x^4 + y^2 - 1 \\ \end{cases}$

$\mathcal{L}_1 = g$

$\nabla f(x,y) = (f_x, f_y) = (y, x)$

$\nabla g(x,y) = (g_x, g_y) = (2x\lambda, 2y\lambda)$

$\begin{cases} y = 2\lambda x \\ x = 4\lambda^2 x \Rightarrow x(1-4\lambda) = 0 \end{cases}$

1) $x = 0 \Rightarrow y = 0$ NO non è sulla circonferenza

2) $(1 - 4\lambda^2) \Rightarrow \lambda^2 = \frac{1}{4} \Rightarrow \begin{cases} \to \lambda_1 = -\frac{1}{2} \\ \to \lambda_2 = \frac{1}{2} \end{cases}$

$\begin{cases} \longrightarrow \\ y = x \\ \lambda = \frac{1}{2} \end{cases} \Rightarrow \begin{cases} x^2 + x^2 - 1 = 0 \Rightarrow 2x^2 = 1 \Rightarrow x^2 = \frac{1}{2} \Rightarrow \\ \end{cases}$

$$\begin{cases} \Rightarrow x = \pm \frac{\sqrt{1}}{2} \quad x = \pm \frac{\sqrt{2}}{2} \\ y = \pm \frac{\sqrt{2}}{2} \\ \lambda = \frac{1}{2} \end{cases}$$

$$P_1 = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \qquad P_2 = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

$$P_3 = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) \qquad P_4 = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$$f(P_1) = \frac{1}{2} \qquad f(P_2) = \frac{1}{2}$$

$$f(P_3) = -\frac{1}{2} \qquad f(P_4) = \frac{-1}{2}$$

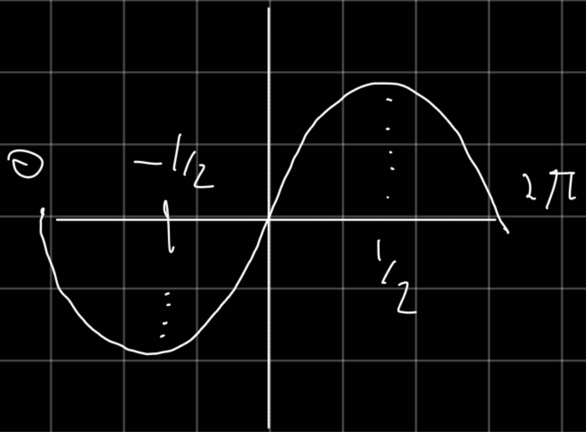
Sono punti di massimo (P_1, P_2) e minimo (P_3, P_4). Se avessi usato un punto solo, altri nascosti

Metodo 2 (Parametrico)

$$\begin{cases} x = X(t) = \cos(t) \\ y = Y(t) = \sin(t) \end{cases} \quad t \in [0, 2\pi]$$

$$h(t) = f(\cos(t), \sin(t)) = \cos(t) \cdot \sin(t) = *$$

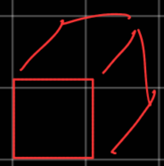
$$* = \frac{1}{2} \sin 2t$$



$$x\left(\frac{1}{2}\right) =$$

$$y\left(\frac{1}{2}\right) =$$

Es. 3) . Scatola a forma



Massimizzo il volume con $A_0 = 12\, m^2$

$$V(x,y,z) = xyz \qquad A_0 = 2xy + 2yz + 2xz$$

$$\frac{A_0}{2} = xy + yz + xz$$

$$\frac{A_0}{2} - xy = z(x+y) \qquad z = \frac{\frac{A_0}{2} - xy}{x+y}$$

$$\tilde{V}(x,y) = \frac{xy\frac{A_0}{2} - xy}{x+y} = \frac{xyA_0 - 2xy}{2(x+y)}$$

$$\tilde{V}_x(x,y) =$$

$$\frac{\frac{d(xyA_0 - x^2y^2)}{dx}\cdot 2(x+y) - xy(A_0 - 2xy)\frac{d(2(x+y))}{dy}}{4(x+y)^2}$$

$$= \frac{(yA_0 - 4y^2x)\overset{1}{\cancel{2}}(x+y) - \overset{1}{\cancel{2}}xy(A_0 - 2xy)}{\overset{2}{\cancel{4}}(x+y)^2} =$$

$$= \frac{xyA_0 - 4x^2y^2 + y^2A_0 - 4y^3x - xyA_0 + 2x^2y^2}{2(x+y)^2} = *$$

$$* = \frac{y^2(A_0 - 2x^2 - 4xy)}{2(x+y)} = \tilde{V}_x$$

$$\tilde{V}_y = \frac{x^2(A_0 - 2y^2 - 4xy)}{2(y+x)^2}$$

$$\begin{cases} \tilde{V}_x = 0 \\ \tilde{V}_y = 0 \end{cases} \Rightarrow \begin{cases} y^2(A_0 - 2x^2 - 4xy) = 0 \\ x^2(A_0 - 2y^2 - 4xy) = 0 \end{cases}$$

$y = 0$

$\rightarrow A_0 - 2x^2 - 4xy = 0$

$\rightarrow$ come la y, stesso la x perché $\neq 0$

$$\begin{cases} A_0 - 2x^2 - 4xy = 0 \\ A_0 - 2y^2 - 4xy = 0 \end{cases}$$

$$\begin{cases} 2x^2 = A_0 - 4xy \\ 2y^2 = A_0 - 4xy \end{cases} \Rightarrow \begin{cases} x^2 = \frac{A_0}{2} - 2xy \\ y^2 = \frac{A_0}{2} - 2xy \end{cases}$$

$$\Downarrow$$

$$x^2 = y^2$$

$$\Downarrow$$

$$x = y \qquad x = -y$$

$x = y$

$$x^2 = \frac{A_0}{2} - 2x^2 \qquad\qquad x^2 = \frac{-A_0}{2}$$

$$x^2 = \frac{A_0}{6} \Rightarrow x = \pm\frac{\sqrt{A_0}}{6}$$

$$P_1 = \left(\frac{\sqrt{A_0}}{6}, \frac{\sqrt{A_0}}{6}\right) \qquad P_2 = \left(-\frac{\sqrt{A_0}}{6}, \frac{\sqrt{A_0}}{6}\right)$$

2) Studio Hessiana

$$\tilde{V}_{xy}(x,y) = \frac{y^2}{2}\frac{d}{dx}\frac{A_0 - 2x^2 - 4xy}{(x+y)^2} =$$

$$= \frac{\frac{y^2}{2}\left(-4x-4y\right)(x+y)^{2} - (2\cancel{x+y})\left(A_0-2x^2-4xy\right)}{(x+y)^3}$$

$$= \frac{y^2}{2}\frac{-4(x+y)^2 - 2\left(A_0 - 2x^2 - 4xy\right)}{(x+y)^3} =$$

$$=$$

$$= -y^2\frac{2y^2 + A_0}{(x+y)^3}$$

$$\tilde{V}_{yy} = -x^2\frac{2x^2 + A_0}{(x+y)^3}$$

$$V_{xy}(x,y) = \frac{1}{2}\frac{d\left(y^2(A_0 - 2x^2 - 4xy)\right)\cdot(x+y^2) - y^2(A_0 - 2x^2 - 4xy)2(x+y)}{(x+y)^4}$$

$$\tilde{V}_{xy} = \frac{xy\left(A_0 - 2x^2 - 6xy - 2y^2\right)}{(x+y)^3}$$

$$\overset{\text{per Teorema Schwarz}}{=} V_{yx}$$

$$\tilde{V}_{xx}(P_1) = -\frac{\sqrt{6A_0}}{6}$$

$$\tilde{V}_{yy}(P_1) = -\frac{\sqrt{6A_0}}{6}$$

$$\tilde{V}_{xy} = -\frac{\sqrt{6A_0}}{12}$$

$$H_{P_1}\begin{pmatrix} -\frac{\sqrt{6A_0}}{6} & -\frac{\sqrt{6A_0}}{12} \\ -\frac{\sqrt{6A_0}}{12} & -\frac{\sqrt{6A_0}}{6} \end{pmatrix}$$

$\mathrm{Det}_H(P_1) = \frac{A_0}{8} > 0 \Rightarrow$ autovalori concordi

$\mathrm{Tr}_H(P_1) = -\frac{\sqrt{6A_0}}{3} < 0 \Rightarrow$ AUTOV. NEG

$\Downarrow$

P_1 è UN MASSIMO

$$H(P_2) = \begin{pmatrix} \frac{\sqrt{6A_0}}{6} & \frac{\sqrt{6A_0}}{12} \\ \frac{\sqrt{6A_0}}{12} & \frac{\sqrt{6A_0}}{6} \end{pmatrix}$$

$$Det_H(P_2) = \frac{A_2}{8} > 0 \Rightarrow P_2 \text{ minimo locale}$$

$$Tr_H(P_2) = \frac{\sqrt{6A_0}}{3} > 0 \Rightarrow$$