X cosa

n pari $f_n(x) \begin{cases} n & x \in [\frac{1}{2} + \frac{1}{n}, 1] \\ 0 & \text{altrove} \end{cases}$

n dispari $f_n(x) = \begin{cases} n & x \in [0, \frac{1}{n}] \\ 0 & \text{altrove} \end{cases}$

Studio conv. puntuale ed uniforme e trovo un I se ∃ dove converge uniformemente.

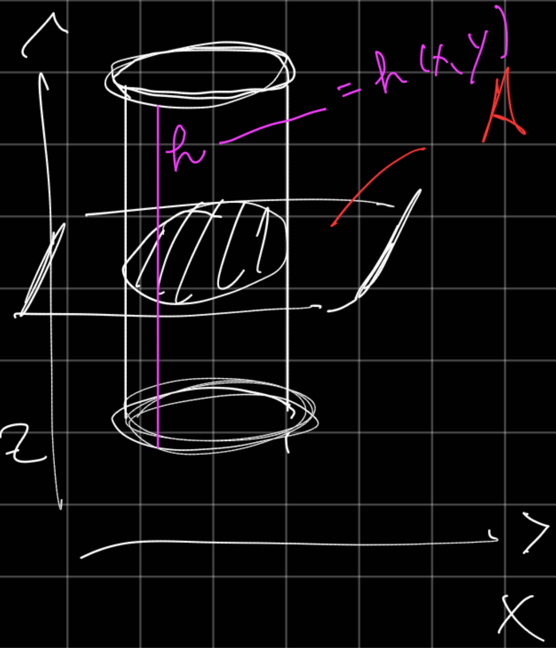
Es. Integrali Multipli:

(1) $V = \{(x, y, z) \in \mathbb{R}^3 : \underbrace{x^2 + y^2 + z^2}_{\text{"sfera"}} \le 4, z \ge 1\}$

Calcolare $\iiint_V x^2 yz \, dx\, dy\, dz$

Metodo 1) Integrale per strati

pezzo "perpendicolare"



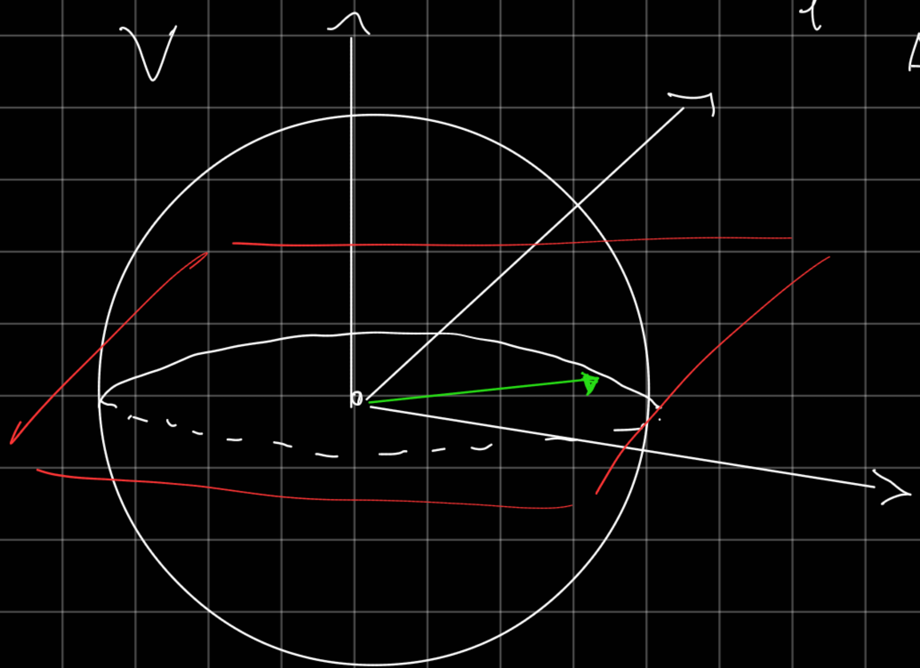
$A = A(z)$

$$\int A(z)\,dz = V$$

$$\iint h(x,y)\,dx\,dy$$

$$z \in [1,2]$$

$$\iiint_V x^2 y z \,dx\,dy\,dz = \int_1^2 dz \iint_{\Delta z} x^2 y z \,dx\,dy$$



in 2d stabilisco e ottengo la sfera

Coord. Polari:

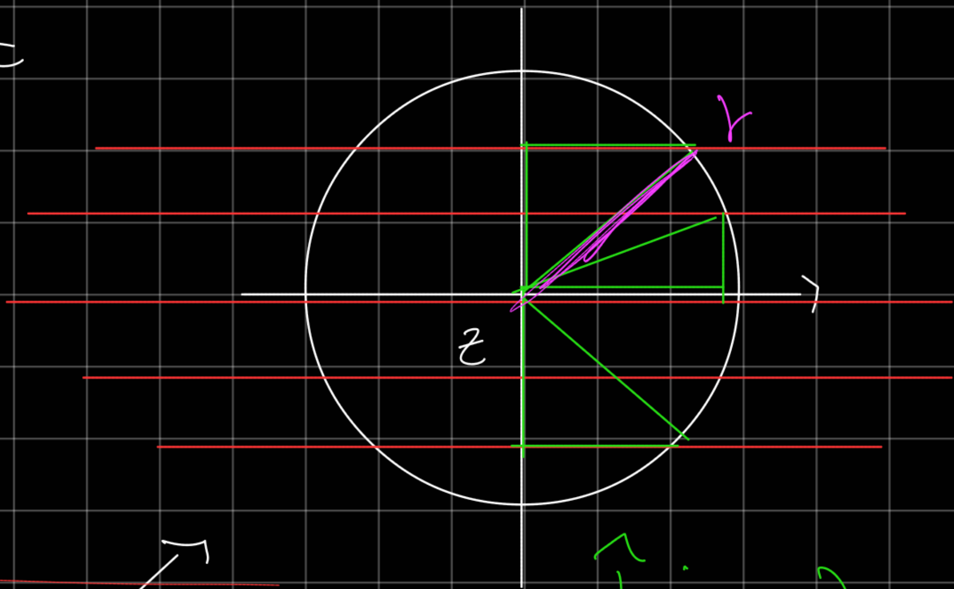
$$\begin{cases} \rho = x^2 + y^2 \\ \vartheta \end{cases}$$

$$\begin{cases} x = \rho\cos\vartheta \\ y = \rho\sin\vartheta \end{cases}$$

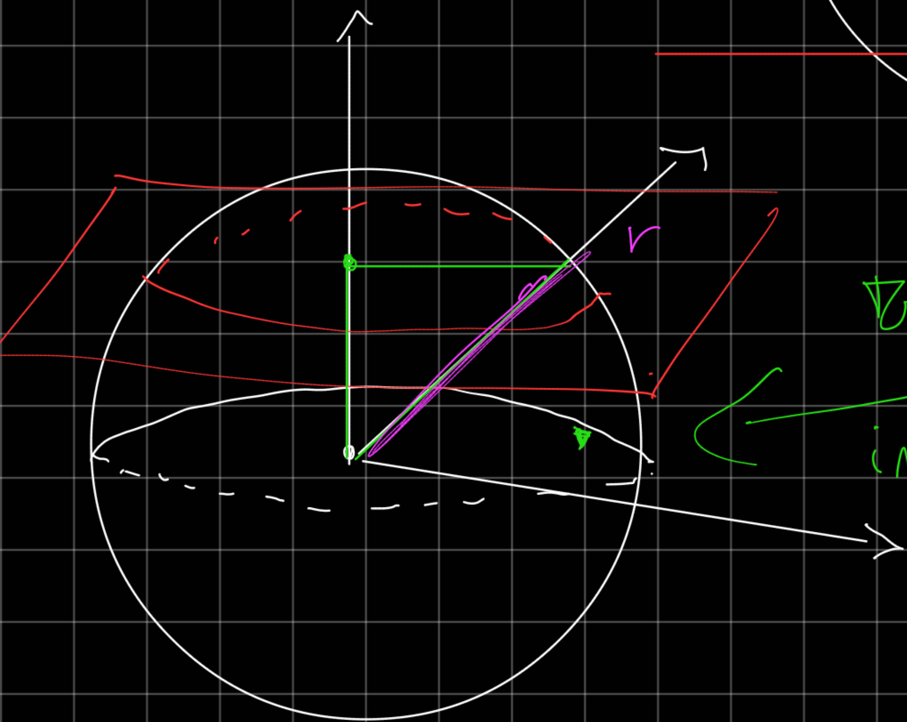
Δ_z cerchio,

Centro $\Delta_z = (0, 0, z)$

Raggio $\Delta_z =$



in 2D



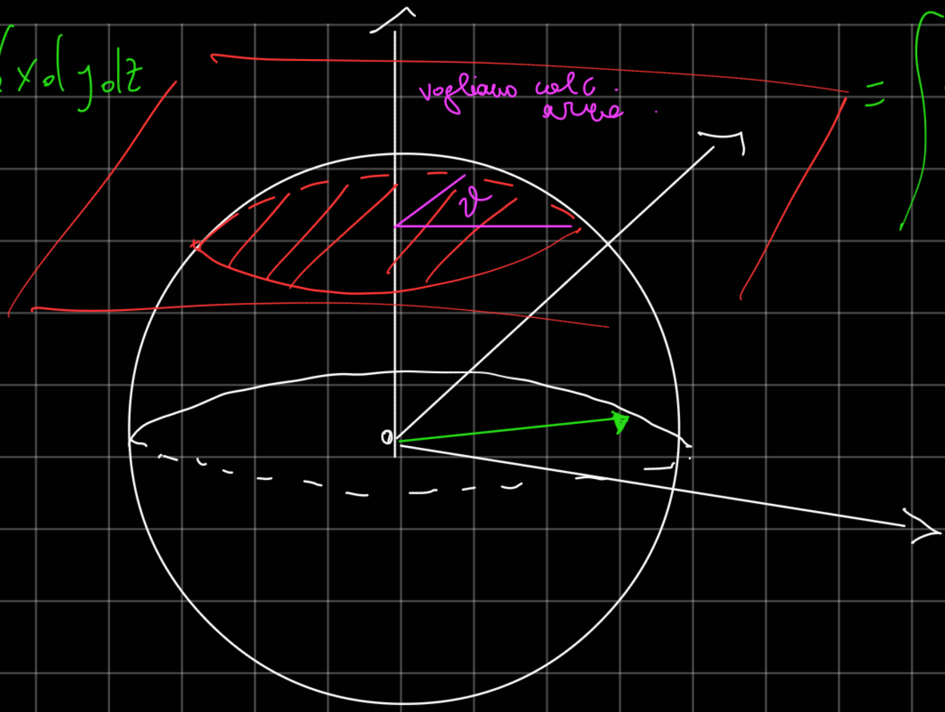
Triangolo in 3D

$\rho(z) = ?$

r costante, è il raggio della circonf.

$$\iint_{\Delta_z} x^2 y \, dx\, dy\, dz = \int_0^{2\pi}\int_0^{\sqrt{4-z^2}} \rho^2 \cos^2\vartheta \, \rho \sin\vartheta \cdot \rho \, d\rho\, d\vartheta$$

$$\iiint_V x^2 y z \, dx\, dy\, dz = \int z \, dz \iint x^2 y z \, dx\, dy$$



$$f(x) \qquad x = g(t)$$

$$\int_A f(x)\, dx \Rightarrow \int_{A^*} f\, g(t)\, d(g(t))$$

$$\frac{d(g(t))}{dt} = g'(t) \cdot dt$$

$$= \int_0^{\sqrt{4-z^2}} \rho^4 \, d\rho \int_0^{2\pi} \cos^2\vartheta \sin\vartheta \, d\vartheta = 0$$

$$\iint_{A_z} x^2 y \, dx\, dy = 0$$

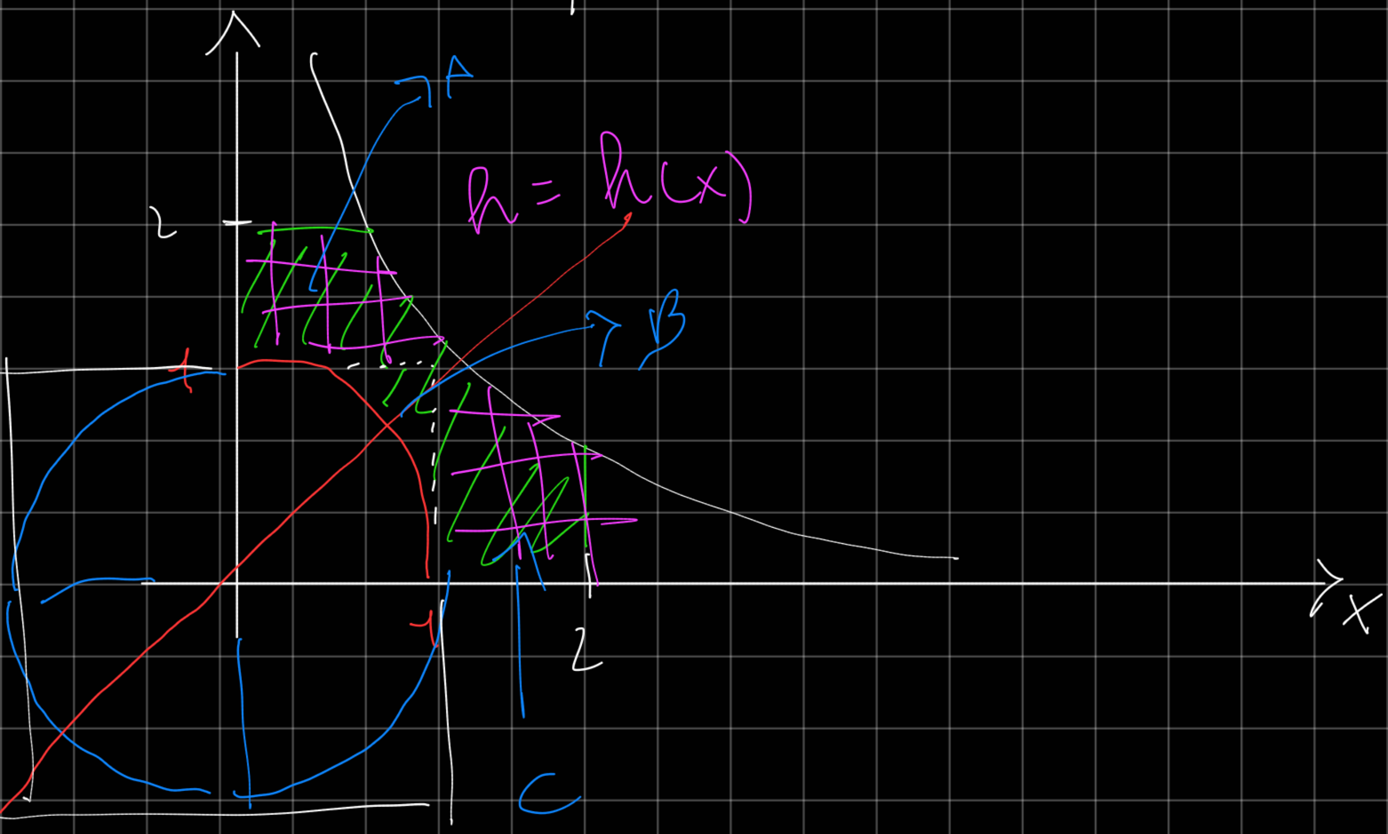
Calcolare $\iiint x^2 y z \, dx\, dy\, dz = 0$

Metodo 2, integrale per fili

Es. integrali multipli.

$$T = \left\{ (x,y) \in \mathbb{R}^2 \text{ t.c. } x^2+y^2 \geq 1,\ xy \leq 1,\ 0 \leq x \leq 2,\ 0 \leq y \leq 2 \right\}$$

Calcolare $\iint_T (x+y)\, dx\, dy$



$$A = \left\{ (x,y) \in \mathbb{R}^2 : 1 \leq y \leq 2,\ 0 \leq x \leq \frac{1}{y} \right\}$$