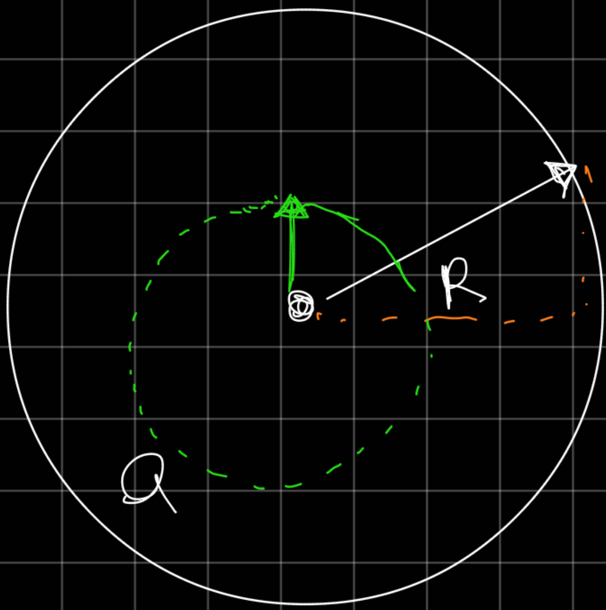
Es. 1



Densità di carica di volume cresce proporzional. dal centro alla superf.

$Q = 10^{-8}\ C$ $\Rightarrow Q_{TOT}$

$R = 10\ cm$

$\rho(r) = K r$

$\rho = 10^{-7}\ \frac{C}{m}$

$\Delta V_{0R} = ?$

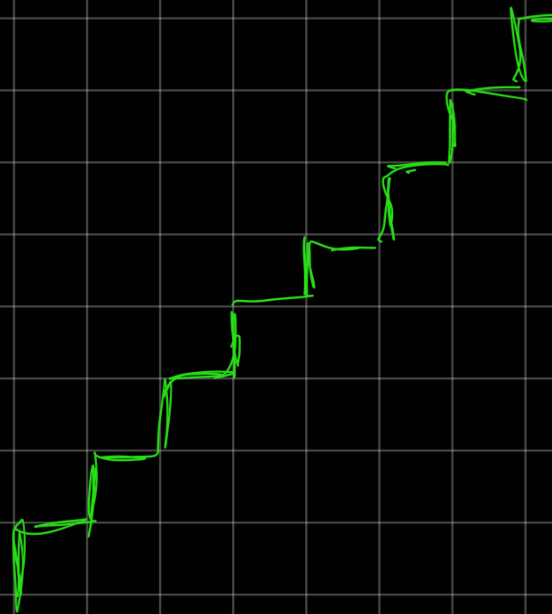
$$\Delta V = \int_A^B E_r \cdot dl = V_B - V_A$$

$$= \int_\infty^A E_r \cdot dr - \int_\infty^B E_r \cdot dr$$

$$E_r = \frac{k_e Q}{\pi r^2 4} \hat{r} \qquad dq = \rho\, dr$$

$$\Delta V = k_e \int_\infty^R \frac{\rho_1 dr\, r}{4 r^2 \pi} - k_e \int_\infty^0 \frac{\rho_2 dr}{4 r^2 \pi} r$$

$$k_e \rho_1 \int_0^R \frac{dr}{\|r^2\|} \hat{i}$$



$$\boxed{E_r(r) = \frac{1}{3}\frac{\rho}{\varepsilon_0} r}$$

d) Calcolare il campo elettrico

Teorema di Gauss, abbiamo simmetria

$$4\pi R^2 \cdot \bar{E}_r(r) = \frac{\iiint k \cdot \rho \, dV}{\varepsilon_0} = \frac{Q_{int}(r)}{\varepsilon_0}$$

$$E_r(r) = \frac{Q_{int}}{\varepsilon_0} \frac{1}{4\pi r^2}$$

$$Q_{int} = \int_0^R k \rho(r) \, 4\pi r^2 \, dr$$

$$\int_0^R \rho(r') 4\pi r'^2 dr' = Q$$

$$\int_0^R r'^3 \, dr^3 = \frac{Q}{k 4\pi}$$

$$k = \frac{Q}{R^4 \pi}$$

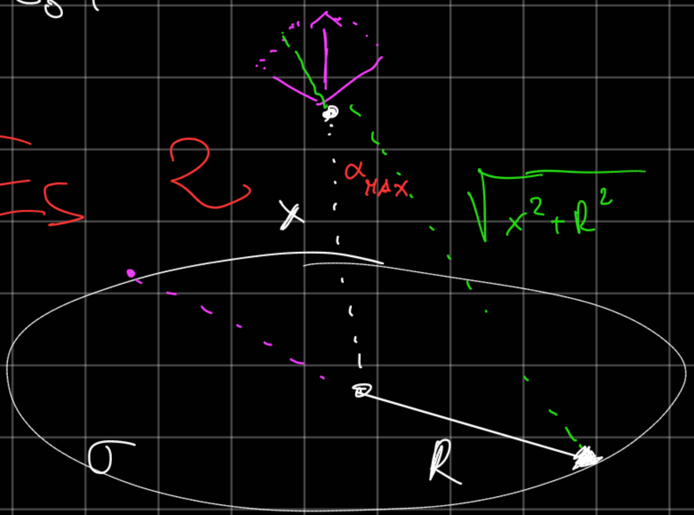
$$\int_0^r \frac{Q}{R^4 \cancel{\pi}} r'^3 \, 4\cancel{\pi} \, dr' = \frac{Q}{R^4} \cancel{4} \cdot \frac{r^4}{\cancel{4}} = \frac{Q}{R^4} r^4$$

$$E_r(r) = \frac{Q}{\varepsilon_0 R^4} r^4 \cdot \frac{1}{4\pi r^2} = \frac{Q r^4}{\varepsilon_0 R^4 r^2 4\pi} = \frac{Q r^2}{\varepsilon_0 R^4 \pi 4}$$

$$V(R) = -\int_0^R \frac{Q r^2}{\varepsilon_0 4\pi R^4} \, dr$$

$$-\frac{Q}{\varepsilon_0 4\pi R^4} \frac{R^3}{3} \qquad \Delta V = \frac{Q}{12\pi\varepsilon_0} \frac{1}{R}$$

Es 2



$E(x)$?

$$dq = \sigma \, ds$$

Sommiamo solo componente $\hat{x}$

$$E_r = \iint_{\sigma} \frac{k_e \, \sigma \, ds}{\left(\sqrt{x^2+R^2}\right)^3} \sqrt{x^2+R^2} \, \text{sen}\alpha_{MAX} = \frac{k_e \, \sigma \, 2\pi r \sqrt{x^2+R^2} \, \text{sen}\alpha}{\left(\sqrt{x^2+R^2}\right)^3}$$

$$d\bar{E} = \frac{\sigma}{2\varepsilon_0} \sin\alpha \, d\alpha$$

$$E(x) = \int_0^{\alpha_{MAX}} \frac{\sigma}{2\varepsilon_0} \sin\alpha \, d\alpha = \frac{\sigma}{2\varepsilon_0}\left[1 - \cos\alpha_{MAX}\right]$$

$$R = x \, tg\,\alpha_{MAX} \qquad \cos\alpha_{MAX} = \sqrt{\frac{1}{1+tg^2\alpha_{MAX}}}$$

$$tg\,\alpha_{max} = \frac{R}{x}$$

$$\sqrt{\frac{1}{1+\frac{R^2}{x^2}}} = \sqrt{\frac{1}{\frac{x^2+R^2}{x^2}}}$$

$$= \sqrt{\frac{x}{\sqrt{x^2+R^2}}}$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]$$

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