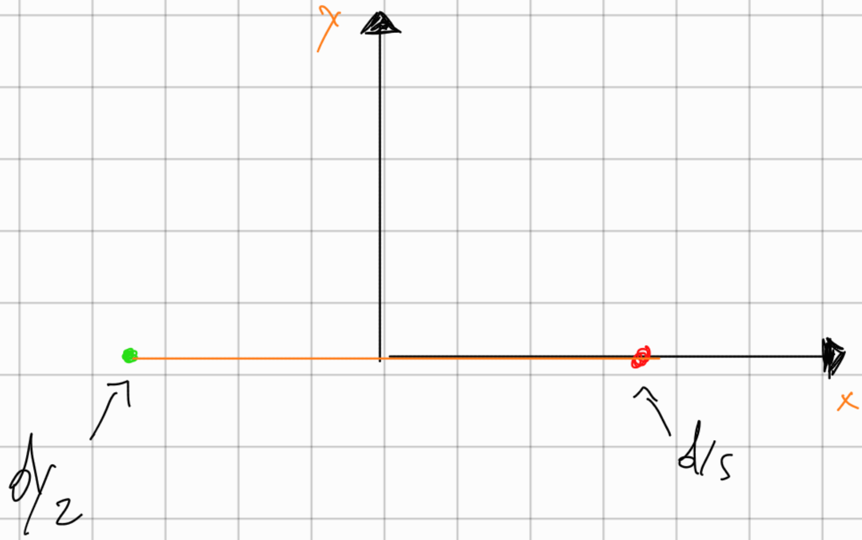
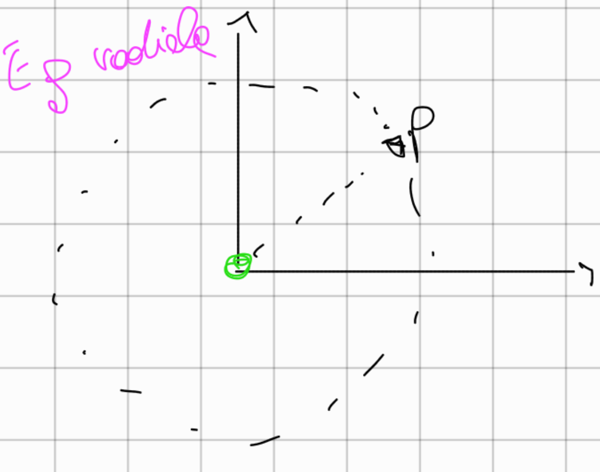
Esercizio 1



uso Gauss e come sup. di Gauss prendo cilindro

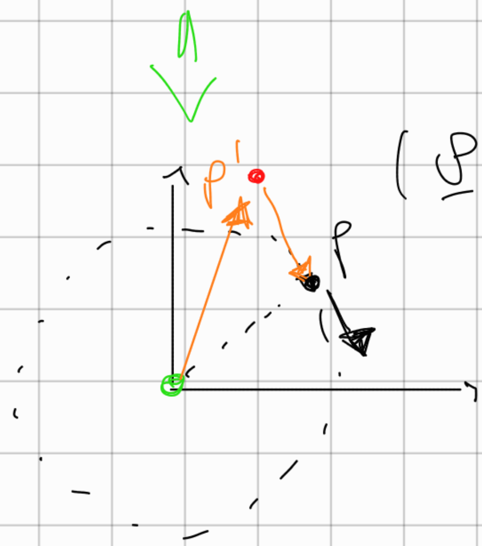
$$E_\rho \, 2\pi r L = \frac{\lambda L}{\varepsilon_0} \Rightarrow \bar{E}_\rho = \frac{1}{2\pi\varepsilon_0}\frac{1}{\rho}\hat{\rho}$$

distanza radiale



campo di se avessi il filo nell'origine

ma io ho due fili che non sono nell'origine



$(\rho - \rho')$

$$\frac{(\rho - \rho')}{|\rho - \rho'|}$$

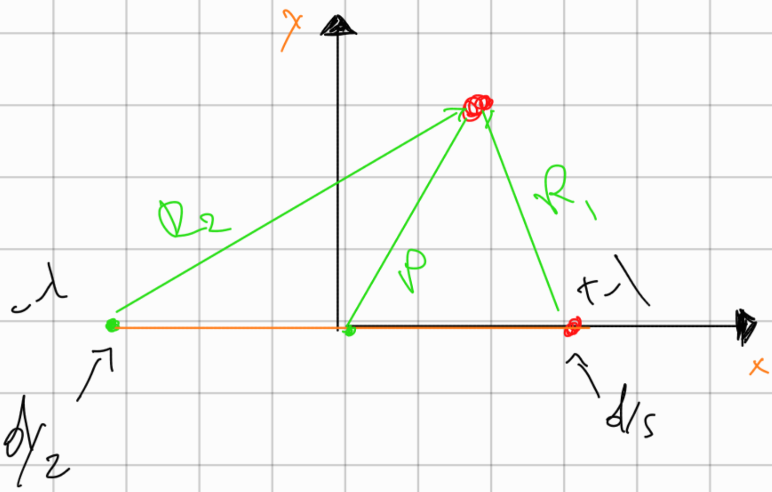
$$\underline{E}(x,y) = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{|\rho-\rho'|^2}(\underline{\rho}-\underline{\rho}')$$

con questa formula posso calcolare campo elettrico dei due fili, usando ρ' per individuare i fili.

$$\rho_1' = \frac{d}{2}\hat{x}$$

$$\rho_2' = -\frac{d}{2}\hat{x}$$

$$\underline{\rho} = x\hat{x} + y\hat{y}$$



$$(\rho-\rho_1') = \left(x-\frac{d}{2}\right)\hat{x} + y\hat{y}$$

$$(\rho-\rho_2') = \left(x+\frac{d}{2}\right)\hat{x} + y\hat{y}$$

$$|\rho-\rho_1'| = \sqrt{\left(x-\frac{d}{2}\right)^2 + y^2}$$

$$|\rho-\rho_2'| = \sqrt{\left(x+\frac{d}{2}\right)^2 + y^2}$$ → 1ª circ.

$$E_1(x,y) = \frac{-1}{2\pi\varepsilon_0}\frac{\lambda}{\left(x-\frac{d}{2}\right)^2+y^2}\left[\left(x-\frac{d}{2}\right)\hat{x} + y\hat{y}\right]$$

$$+\ E_2(x,y) = \frac{-1}{2\pi\varepsilon_0}\frac{\lambda}{\left(x+\frac{d}{2}\right)^2+y^2}\left[\left(x+\frac{d}{2}\right)\hat{x} + y\hat{y}\right]$$

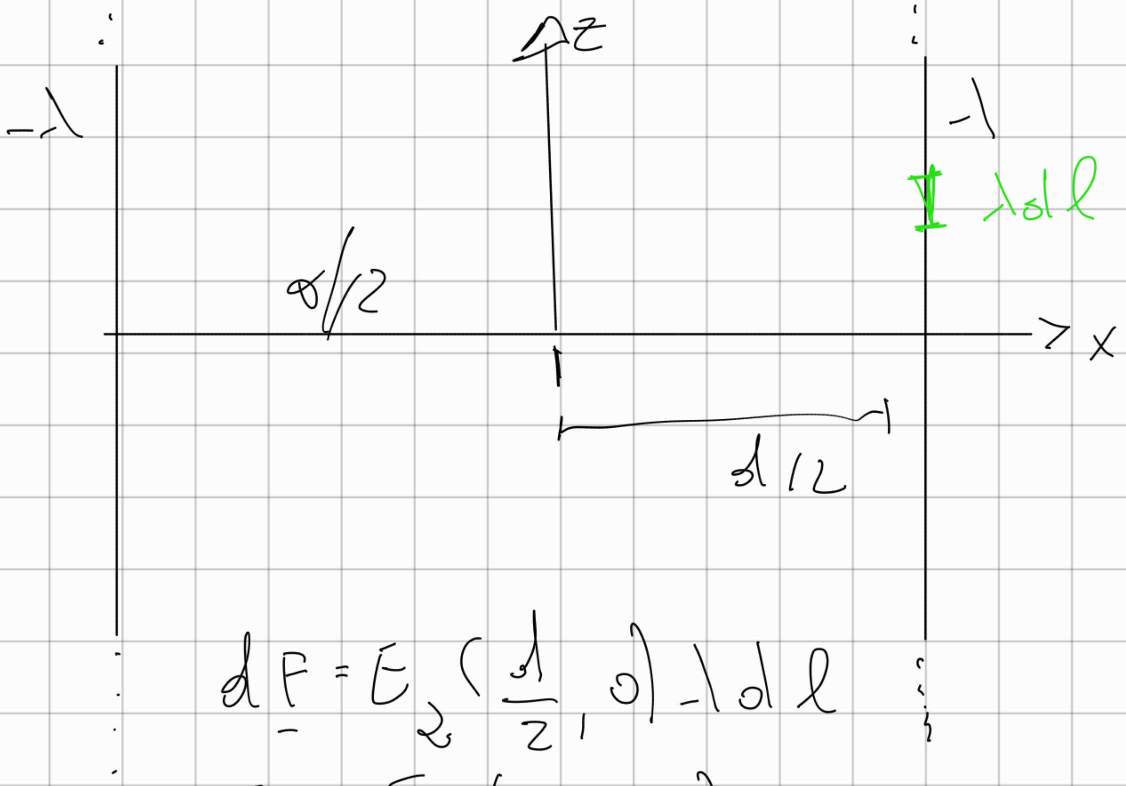
1ª circonferenza

$$\begin{cases} \left(x-\frac{d}{2}\right)^2 + y^2 = R_1^2 \\ \left(x+\frac{d}{2}\right)^2 + y^2 = R_2^2 \end{cases} \Rightarrow \begin{matrix} x^2 + \frac{d^2}{4} + 2xd + y^2 = R_1^2 \\ x^2 + \frac{d^2}{4} - 2xd + y^2 = R_2^2 \end{matrix}$$

$$4xd = R_1^2 - R_2^2 \qquad x_1 = \left(\frac{R_1^2 - R_2^2}{4d}\right)$$

$$y^2 = R_1^2 - 2xd - \frac{d}{4} - x^2 \Rightarrow y_1 = \sqrt{R_1^2 - \left(x_1 - \frac{d}{2}\right)^2} = 0{,}17\ m$$

Forza che agisce su un tratto di filo?



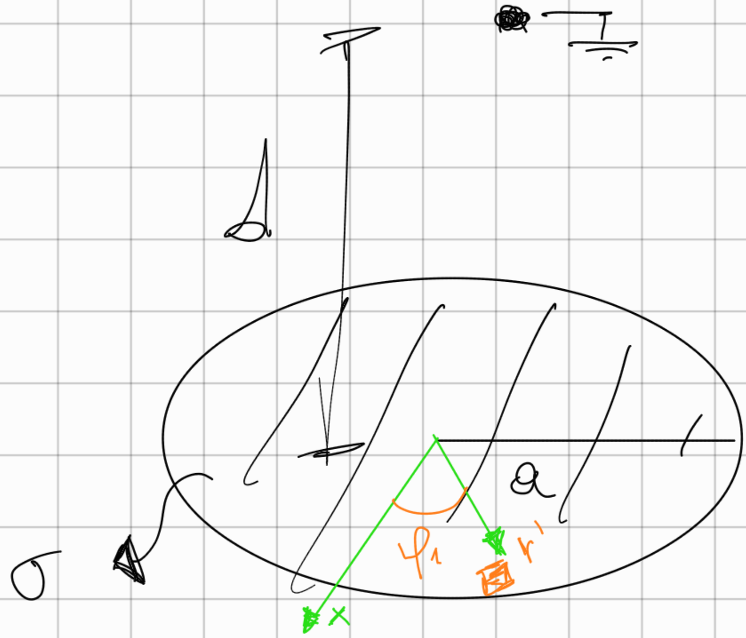
$$d\underline{F} = \underline{E}_2\left(\frac{d}{2}, 0\right) \lambda \, dl$$

$$\underline{F}_m = \underline{E}_2\left(\frac{d}{2}, 0\right) \lambda$$

$$\underline{E}_2\left(\frac{d}{2}, 0\right) = -\frac{1}{2\pi\varepsilon_0} \frac{\lambda}{d^2} d\hat{x} = \frac{-1}{2\pi\varepsilon_0 d} \lambda (\hat{x})$$

$$F_n = -\frac{1}{2\pi\varepsilon_0}\frac{\lambda^2}{d}\hat{x}$$

Disco di 1 mm con carica uniforme



$R = 1\,mm$

$a = 10\,cm$

$d = 30\,cm$

$\sigma = 10^{-11}\,C/m^2$

piccola approssimazione, e la sfera è a massa

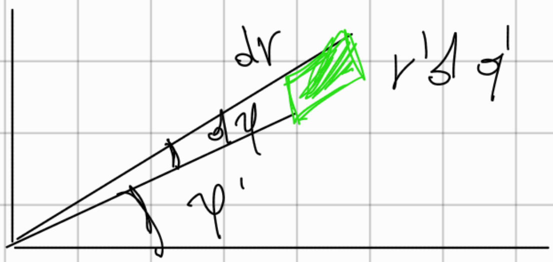
piastra induce spostamento elettroni piccola sfera, ma carica positiva va a terra

ESSENDO LA SFERA PICCOLA, APPROSSIMO la distribuzione a UNIFORME

$$V_s(R) = k_e\frac{Q}{R}$$

potenziale che vede la sfera

$$ds' = r'\,d\varphi'\,dr'$$



$$V_D(\underline{r}) = \iint_S \frac{k_e \, dq}{|r - r'|} = k_e \int_0^{2\pi}\int_0^{a} \frac{\sigma r' \, d\varphi' \, dr'}{\sqrt{(r')^2 + d^2}} =$$

$$r = (0, 0, d) = d\hat{z}$$

$$r' = r' \cos\varphi' \hat{x} + r' \sin\varphi' \hat{y}$$

$$|r - r'| = \sqrt{(r')^2 + d^2}$$

$$= k_e \sigma 2\pi \int_0^a \frac{r'}{\sqrt{(r')^2 + d^2}} dr' =$$

$$= k_e \sigma 2\pi \left[\sqrt{(r')^2 + d^2} \right]_0^a =$$

$$\boxed{= k_e \sigma 2\pi \left[\sqrt{a^2 + d^2} - d \right]}$$

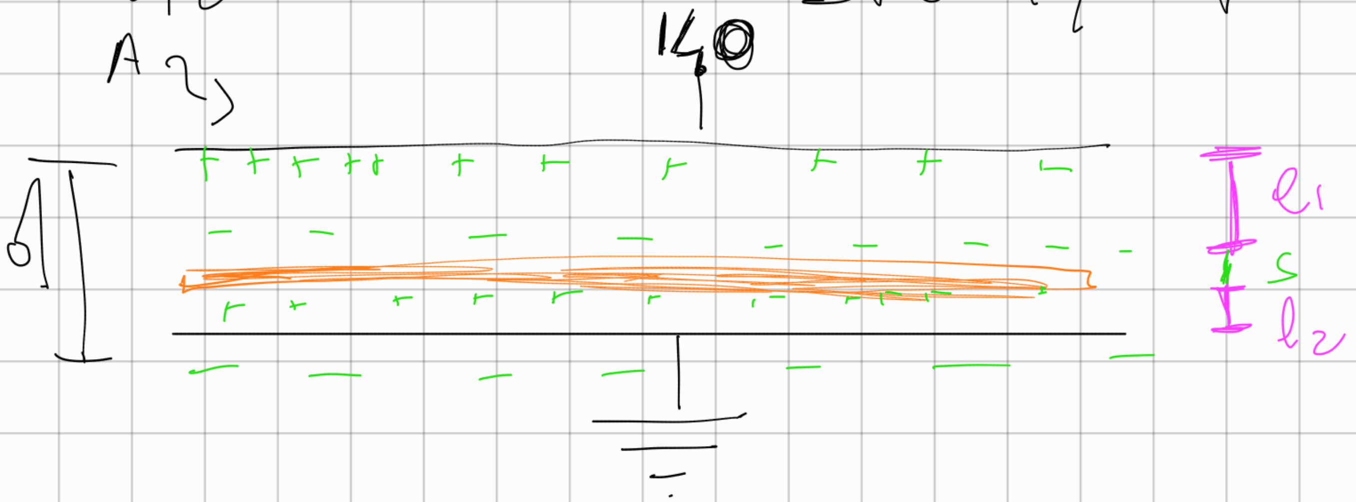
$$V(d) = V_S(R) + V_D(d) = k_e \frac{Q}{R} + k_e \sigma 2\pi \left[\sqrt{a^2 + d^2} - d \right] = 0$$

perché sfera a terra

$$Q = -\sigma 2\pi \left[\sqrt{a^2 + d^2} - d \right] R$$

carica indotta su sferettina

$A = 1{,}5\ m^2 \qquad d = 2\ cm \qquad \Delta V_0 = 140\ V$



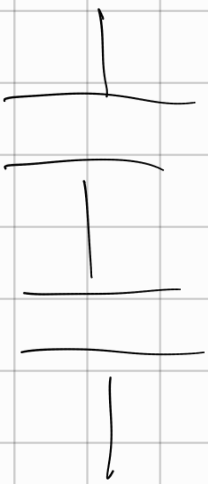
$$C_0 = \frac{\varepsilon_0 A}{d} = 664\ pF$$

$$Q_0 = C_0 \Delta V \cong 92.9\ nC$$

una volta carico, si stacca il generatore di tensione, e viene inserito a caso un conduttore

Ho allora 2 condensatori C_1 e C_2

$$\boxed{l_1 + l_2 + s = d}$$



$$C_1 = \frac{\varepsilon_0 A}{l_1}$$

$$C_2 = \frac{\varepsilon_0 A}{l_2}$$

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{l_1}{\varepsilon_0 A} + \frac{l_2}{\varepsilon_0 A} = \frac{l_1 + l_2}{\varepsilon_0 A}$$

$s \rightarrow d - s$

$$= \frac{d - s}{\varepsilon_0 A}$$

non è rilevante dove vado a mettere il conduttore, la capacità totale dipende solo da $d - s$

$$\Delta C = C_T - C_0 = \frac{\varepsilon_0 A}{d - s} - \frac{\varepsilon_0 A}{d} =$$

$$= \varepsilon_0 \left(\frac{1}{d-s} - \frac{1}{d} \right) = \varepsilon_0 A \, \frac{\cancel{d} - \cancel{d} + s}{(d-s)\, d}$$

$$\Delta C = C_0 \left(\frac{s}{d-s} \right) \simeq 117\, pF$$

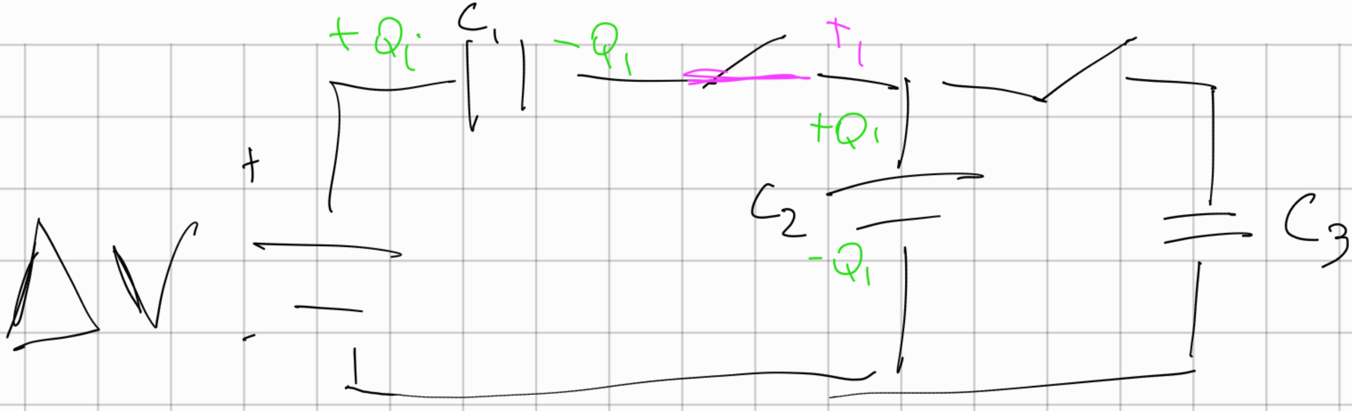
$$\Delta V_N = \frac{Q}{C_T} = \frac{Q}{\varepsilon_0 A} \, \frac{d\,(d-s)}{d} =$$

nuova diff di pot

Trucco

$$= \frac{Q}{C_0} \left(\frac{d-s}{d} \right) = \Delta V_0 \left(\underbrace{\frac{d-s}{d}}_{} \right) = 119\, V$$

0,85



$C_1 = 1\mu F \qquad C_2 = 2C_1 \qquad C_3 = 4C_1$

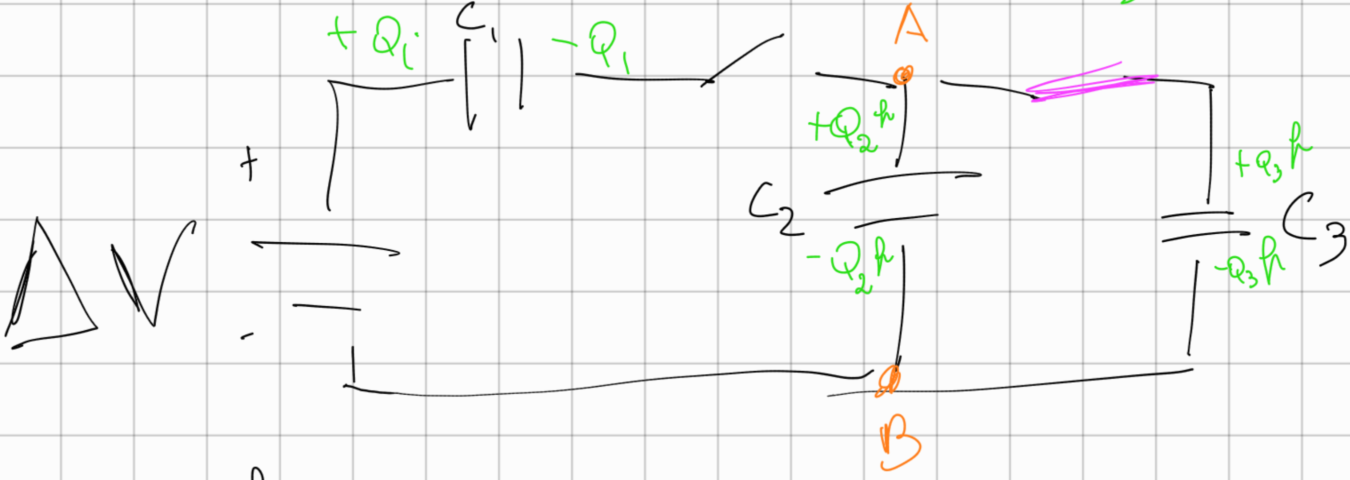
$\Delta V_0 = 50$

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{10^{-6}} + \frac{1}{2\,10^{-6}} = \frac{1}{10^{-6}}\left(1+\frac{1}{2}\right)$$

$$= \frac{3}{2}10^{6}$$

$$C_T = \frac{2}{3}10^{-6} \cdot 50 \simeq 3.33 \cdot 10^{-5}$$

una parte della carica di C_2 andrà su C_3



$Q_i = Q_2^f + Q_3^f$

$$\Delta V_{AB} = \frac{Q_1}{C_2 + C_3} = 5{,}55\,V$$

$C_{parallelo} = C_2 + C_3$

ora che so la ΔV, posso calcolare le Q_2^f e Q_3^f

$$Q_2^f = C_2 \Delta V_{AB} = \frac{C_2}{C_2 + C_3} \, 1{,}11 \cdot 10^{-5} C$$

$$Q_3^f = C_3 \Delta V_{AB} = \frac{C_3}{C_2 C_3} = 2{,}22 \cdot 10^{-5} C$$