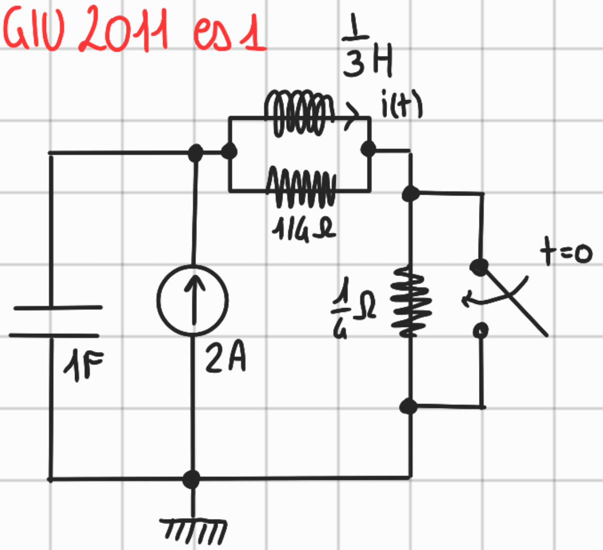
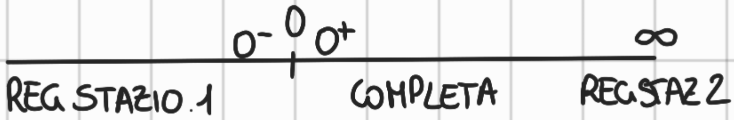
27 GIU 2011 es 1



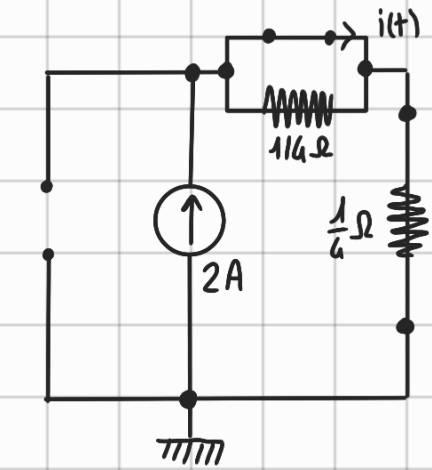
Determina i(t) risposta completa t≥0



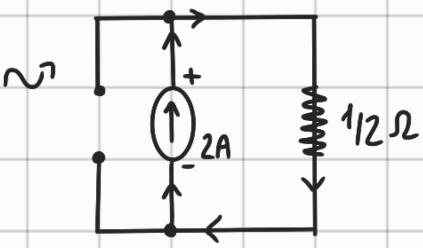
$i(0^-) = i(0^+)$

$V_c(0^-) = V_c(0^+)$

$t = 0^-$

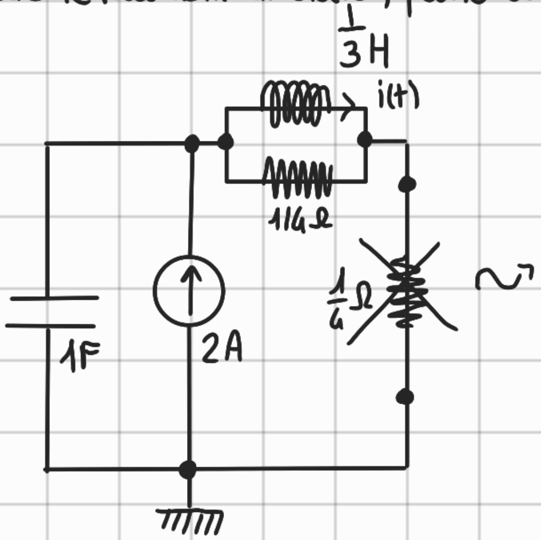


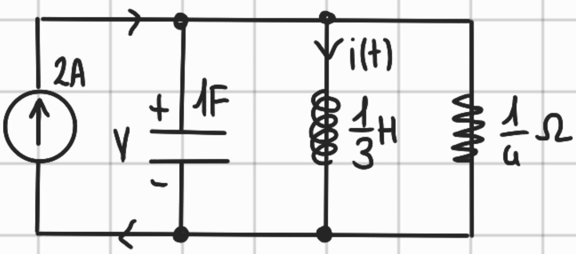
quindi $i(0^-) = 2A = i(0^+)$



$V_c(0^-) = 0$

trovate le variabili di stato, passo a $t = \infty$





RLC //

$$\alpha_p = \frac{G_p}{2C} = \frac{4}{2} = 2$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1/3}} = \sqrt{3}$$

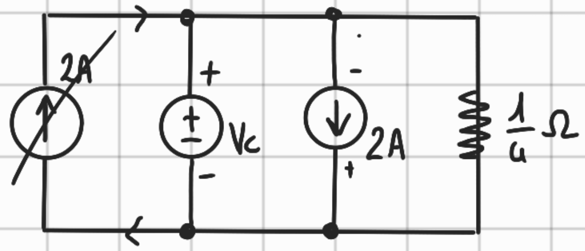
$\rightsquigarrow$ $\alpha_p > \omega_0$ caso sovrasmorzato radici $S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -2 \pm \sqrt{4-3} = -2 \pm 1 = \begin{cases} -1 \\ -3 \end{cases}$

$$i(t)_{TC} = K_1 e^{S_1 t} + K_2 e^{S_2 t} = K_1 e^{-t} + K_2 e^{-3t}$$

$$C = TC + \underbrace{\text{regime 2}}_{=2A}$$

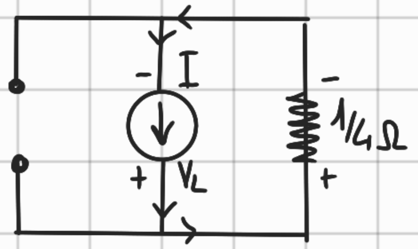
$$i(t) = K_1 e^{-t} + K_2 e^{-3t} + 2$$

Ora devo trovare K_1 e K_2, $\Rightarrow$ studio a 0^+



$I(0^+) = 2\,A$

$\Downarrow$



$$V_L(0^+) = L \frac{di(0^+)}{dt}$$

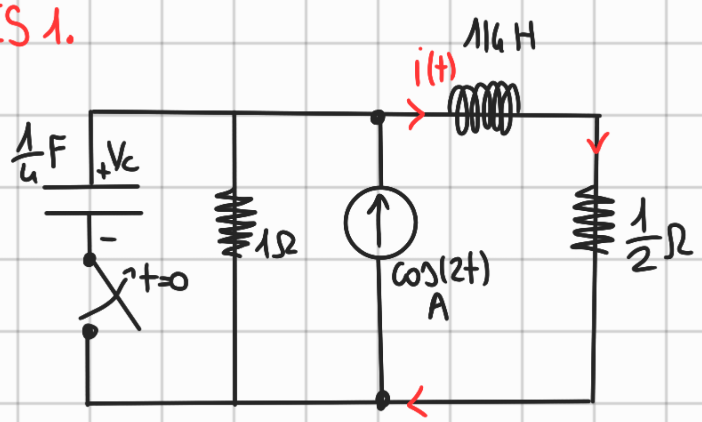
$$V_L = \frac{1}{4} \cdot 2 \qquad \frac{1}{3} \frac{di(0^+)}{dt} = \frac{1}{2}$$

$$\Rightarrow \begin{cases} i(t) = K_1 e^{-t} + K_2 e^{-3t} + 2 \\ i(0^+) = 2 \\ \frac{di(0^+)}{dt} = 3/2 \end{cases}$$

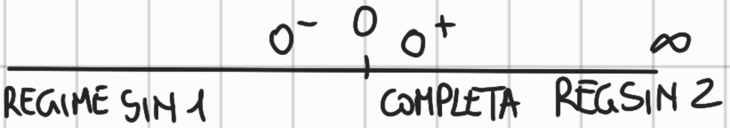
$$\begin{cases} \cancel{2} = K_1 + K_2 + \cancel{2} \\ 3/2 = -K_1 - 3K_2 \end{cases} \quad \begin{cases} K_1 = -K_2 = 3/4 \\ 3/2 = K_2 - 3K_2 \Rightarrow K_2 = -3/4 \end{cases}$$

$$\Rightarrow i(t) = +\frac{3}{4} e^{-t} - \frac{3}{4} e^{-3t} + 2$$

ES 1.



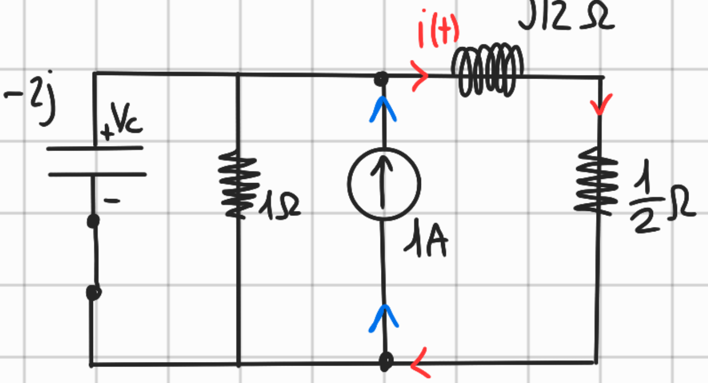
Determina i(t) completa



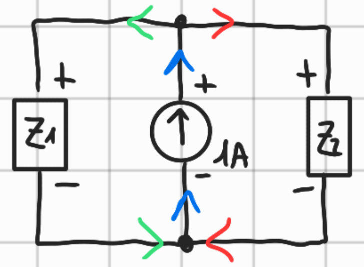
$i_L(0^-) = i_L(0^+)$

$V_c(0^-) = V_c(0^+)$

$t = 0^-$



semplifico



$$\begin{array}{l} K_I \Rightarrow \\ KV \\ \circlearrowright \end{array} \left| \begin{array}{l} 1 = I(0^-) + I_1 \\ Z_2 I(0^-) - V_g = 0 \\ V_g = Z_1 I_1 \end{array} \right.$$

$$Z_1 = \frac{4-2j}{5} \qquad Z_2 = \frac{1+j}{2}$$

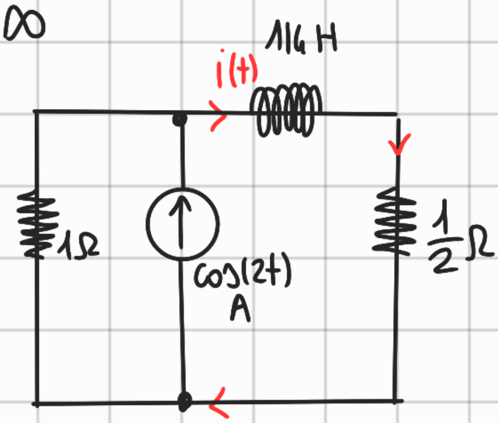
$$I(0^-) = \frac{\frac{1}{Z_2}}{\frac{1}{Z_1} + \frac{1}{Z_2}} = \frac{2}{1+j} \cdot \frac{1}{\frac{5}{4-2j} + \frac{2}{1+j}} = \frac{10}{17} - \frac{6i}{17}$$

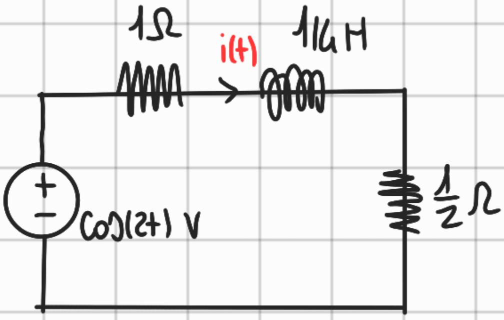
$$i(t) = Re[\bar{I}]$$

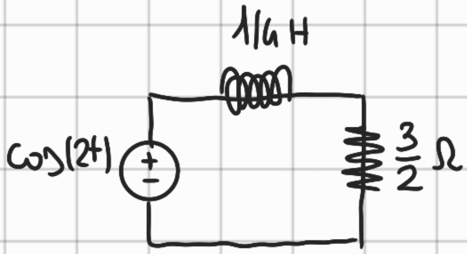
$$i(0^-) = \frac{10}{17} \qquad \tau = \frac{L}{R} = \frac{1/4}{\frac{1}{2}+1} = \frac{1}{6}$$

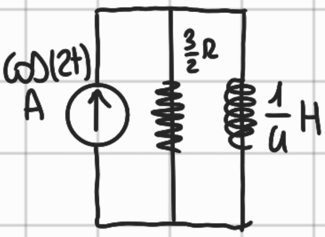
$$\boxed{I_{lib}(t) = \frac{10}{17} e^{-6t} \quad A}$$

$t=\infty$

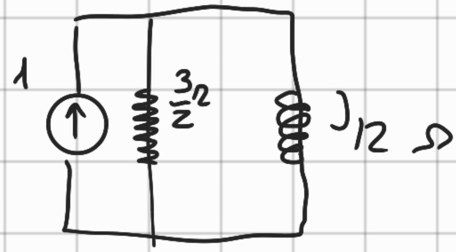








$\Rightarrow$



$V = RI =$

$$i(t) = -\frac{1}{\sqrt{\left(\frac{3}{2}\right)^2 + 2^2\left(\frac{1}{4}\right)^2}} \cos\left(\arctan\left(\frac{2\frac{1}{4}}{3/2}\right)\right) e^{-t\frac{3}{2}\cdot 4} + \frac{1}{\sqrt{\left(\frac{3}{2}\right)^2 + 2^2\frac{1}{4^2}}} \cos\left(2t - \arctan\left(\frac{\omega L}{R}\right)\right)$$

$$= -\frac{2}{\sqrt{10}}\, 0{,}94 + \frac{2}{\sqrt{10}} \cos(2t + 18^\circ)$$

$$= -0{,}6\, e^{-6t} + 0{,}632 \cos\left(2t + 18^\circ\right)$$

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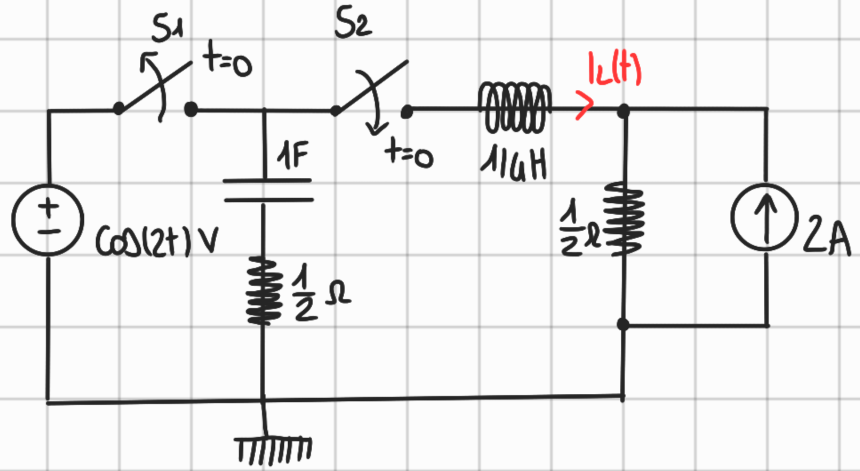
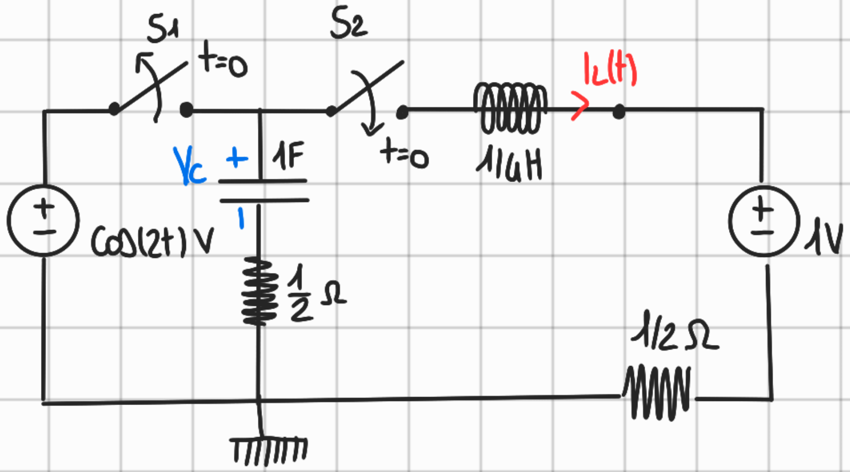


Determina $I_L(t)$

Faccio una semplificazione:

$V = RI$



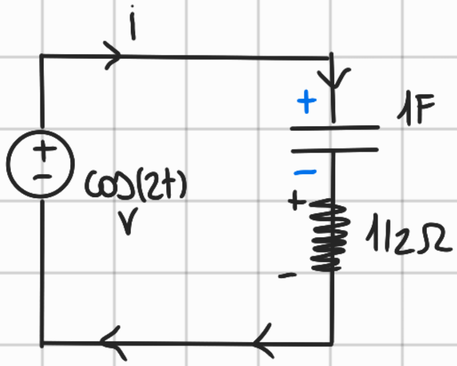
0^- 0 0^+

REGIME SINUSOIDALE | COMPLETA | REGIME STAZIONAR

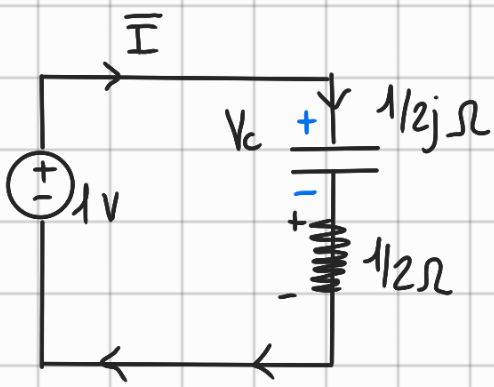
$V(0^-) = V(0^+)$

$I_L(0^-) = I_L(0^+)$

• Analisi a 0^-



FASORI ⇝



trovo $\bar{I} = \frac{V}{Z} = \frac{1}{\frac{1}{2j}+\frac{1}{2}} = \frac{2j\,(1-j)}{(1+j)(1-j)} = \frac{2j+2}{1+1} = 1+j$ A

KV: $V_c + V_R - V = 0$

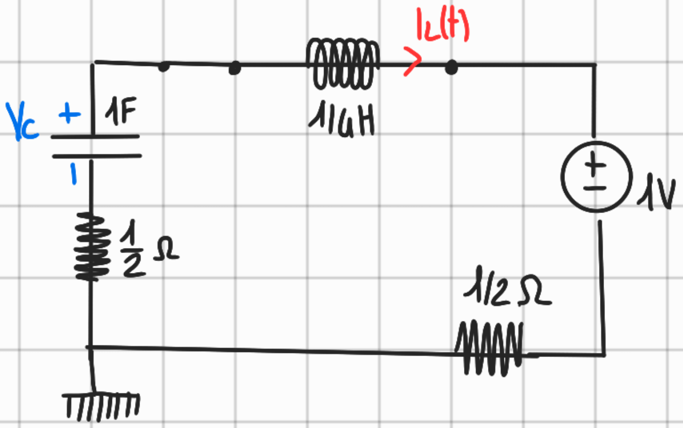
$\Rightarrow V_c = V - V_R = 1 - \frac{1}{2}(1+j) = \frac{2-1-j}{2} = \frac{1-j}{2}$

$Re = \frac{1}{2}$ $\qquad V_c(0^-) = \frac{1}{2}$ V $= V_c(0^+)$

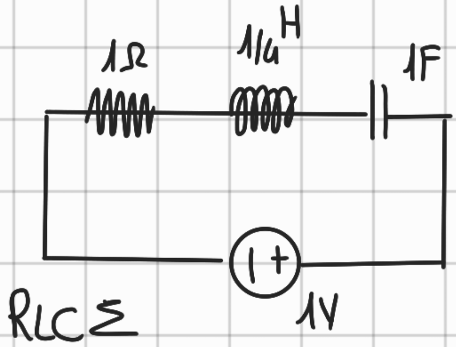
quindi $V_C(0^-) = V_C(0^+) = 1/2\ V$

$I_L(0^-) = I_L(0^+) = 0\ A$

Ora studio a $t=\infty$ ho un REGIME STAZIONARIO



semplifico $\rightsquigarrow$



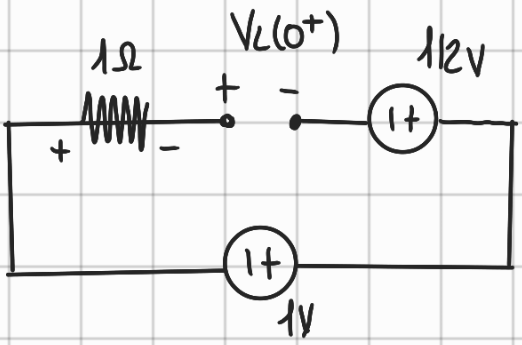
RLC Σ

$$\omega_0 = \frac{1}{\sqrt{LC}} = 2 \qquad \alpha_s = \frac{R_s}{2L} = \frac{1}{2\cdot\frac{1}{4}} = 2$$

$\omega_0 = \alpha_s$ CRITICAMENTE SMORZATO $\quad s_{1,2} = -2$

$$I_L(t) = K_1 e^{-2t} + K_2 t e^{2t} + \underbrace{0}_{\text{REGIME}}$$

Studio a 0^+



$$-1/2 + 1 + L\frac{d}{dt} I_L(t) =$$

$$\cancel{\frac{1}{4}}\frac{dI_L(t)}{dt} = \left(\frac{1}{2} - 1\right)\cdot 4 = -2$$

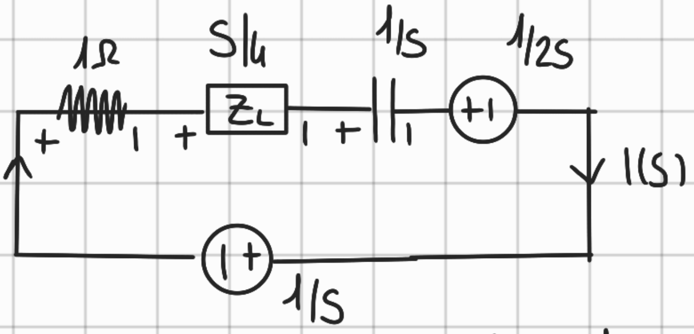
$$\begin{cases} I_L(t) = K_1 e^{-2t} + K_2 t e^{-2t} \\ I_L(0^+) = 0 \\ \dfrac{dI_L(0^+)}{dt} = -2 \end{cases}$$

- $K_1 = 0$
- $K_2 = -2$

$$I_L(t) = -2te^{2t}$$

$$-2 = K_1 \cdot \cancel{2}e^{2t} + K_2 \cancel{e}^{2t} + K_2 t \cancel{2}e^{2t} \qquad K_2 = -2$$

Ora verifichiamo con Laplace



KV $\quad RI + Z_L I + Z_C I + \frac{1}{2S} + \frac{1}{S} = 0$

$$I(S) + \frac{S}{4} I(S) + \frac{1}{S} I(S) + \frac{1}{2S} + \frac{1}{S} = 0$$

$$I(S) = -\frac{2}{(S+2)^2} = -\frac{2}{(S+2)(S+2)} = -\left(\frac{1}{S+2} + \frac{1}{S+2}\right)$$

$$= -\frac{1}{S+2} - \frac{1}{S+2} =$$

$$= -e^{-2t} - e^{-2t} = -2e^{-2t}\, \mathbb{1}(t)$$

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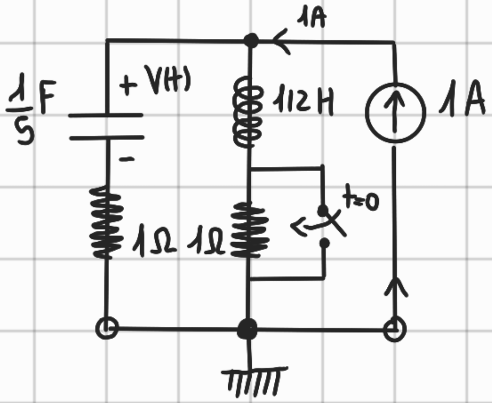
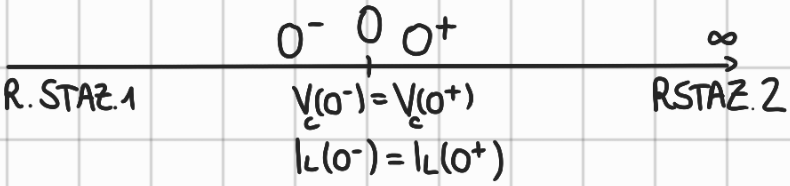
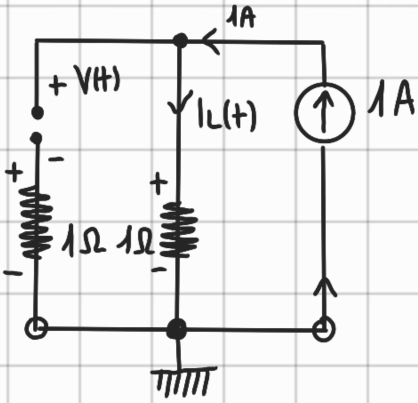


Determina v(t) t≥0 risposta completa
Verificare poi con Laplace



• $t=0^-$ trovo le variabili di stato



REGIME STAZIONARIO C → CA L → CC

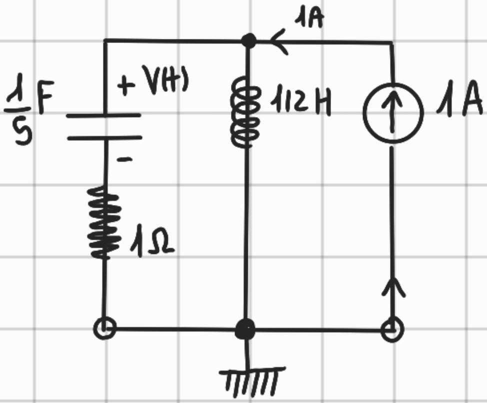
su C.a. non passa corrente quindi

$I_L(0^-) = 1\,A = I_L(0^+)$

KV↻ $R I_L(t) - V(t) = 0$

$V(t) = R I_L(t)$ $V_c(0^-) = 1\,V = V_c(0^+)$

• Studio a $t=\infty$ (R.STAZ 2)



il resistore si scarica e si annulla

Ho un "RLC ||"

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{2}\cdot\frac{1}{5}}} = \sqrt{10} \sim 3{,}16$$

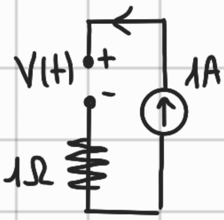
$$\alpha_s = \frac{R_s}{2L} = \frac{1}{2\frac{1}{2}} = 1$$

$\alpha_s < \omega_0$

SOTTOSMORZATO

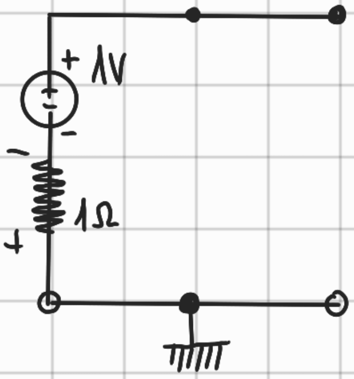
$$S_{12} = -1 \pm \sqrt{(\sqrt{10})^2 - 1^2}\, j = -1 \pm 3j$$

$$V(t) = K_1 e^{-t}\cos(3t) + K_2 e^{-t}\operatorname{sen}(3t) + V_{RS2}$$



$V_c = RI = 0$

• Studio a 0^+



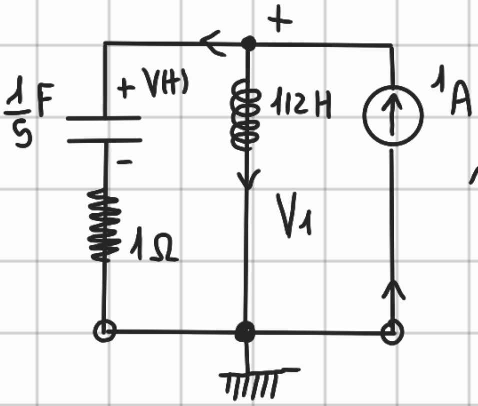
$$dV(t)/dt = 0$$

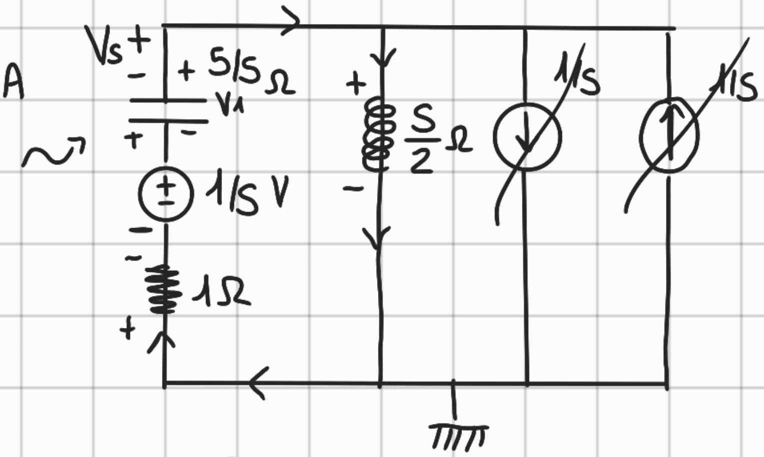
Quindi
$$\begin{cases} V(t) = K_1 e^{-t}\cos(3t) + K_2 e^{-t}\operatorname{sen}(3t) \\ V(0^+) = 1 \quad V \\ dV(0^+)/dt = 0 \quad V/s \end{cases}$$

$$1 = K_1 \qquad K_1 = 1$$
$$0 = K_1\left(-e^{-t}\cos(3t) - e^{-t}3\operatorname{sen}(3t)\right) + K_2\left(-e^{-t}\operatorname{sen}(3t) + 3e^{-t}\cos(3t)\right)$$
$$0 = -1 + K_2 3 \qquad K_2 = 1/3$$

$$V_c(t) = e^{-t}\cos(3t) + \frac{1}{3}e^{-t}\operatorname{sen}(3t)$$

Ora Laplace





$$V(s) - V_1 - \frac{1}{s} = 0 \qquad V_1 = \frac{1}{s}\,\frac{5/s}{\frac{5}{s} + 1 + \frac{s}{2}}$$

$$\hookrightarrow V(s) = V_1 + \frac{1}{s} = \frac{s+2}{s^2+2s+10} = \frac{A}{s-(-1+j3)} + \frac{A^*}{s-(-1-j3)} \quad \checkmark$$

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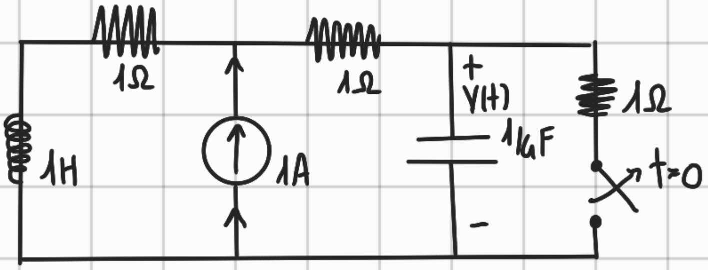


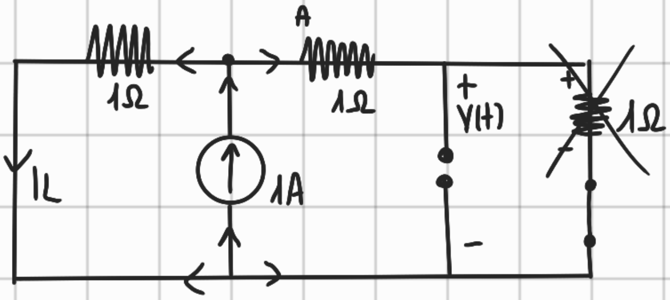
Determina V(t) t≥0 completa
Verifica con LaPlace

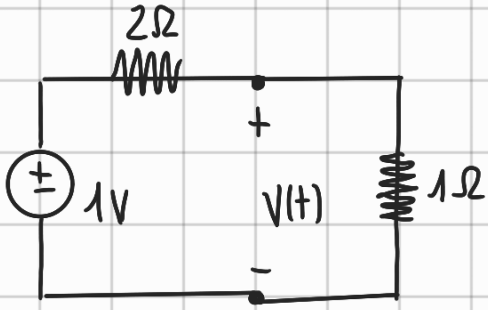
0^- 0 0^+ ∞

RS1 RS2

$V_c(0^-) = V_c(0^+)$
$I_L(0^-) = I_L(0^+)$

• $t=0^-$ trovo le var di stato



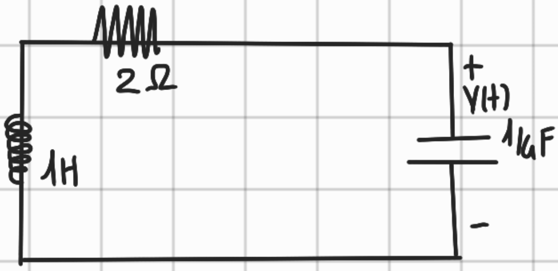


$$V(t) = \frac{\frac{1}{2}}{\frac{1}{2}+1} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{2}\cdot\frac{2}{3} = \frac{1}{3}\ V$$

$V(0^-) = 1/3\ V = V(0^+)$

$I_L(0^-) = \frac{1}{\frac{1}{2}+1} = 2/3\ A = I_L(0^+)$

• $t=\infty$ serie



$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}}} = 2$$

$$\alpha_s = \frac{R_s}{2L} = \frac{2}{2} = 1$$

$\alpha_s < \omega_0$

SOTTOSMORZATO

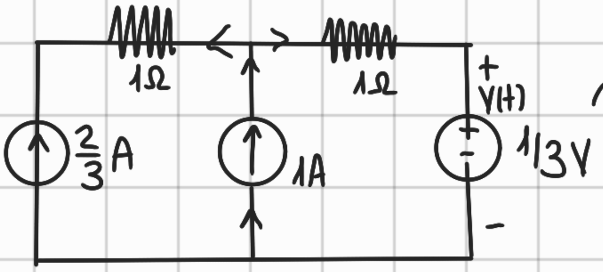
$$S_{12} = -1 \pm \sqrt{2^2-1^2}\,j = -1 \pm \sqrt{4-1}\,j = -1 \pm \sqrt{3}\,j$$

$$V(t) = K_1 e^{-t}\cos(\sqrt{3}t) + K_2 e^{-t}\sin(\sqrt{3}t) + RS2$$

↓

$V_c(t) = 1$

• Studio a 0^+ per trovare $dv(t)/dt$



$$i(t) = C\frac{dV(t)}{dt}$$

$$1 = \frac{2}{3} + \frac{1}{4}\frac{dV(t)}{dt} \qquad \frac{1}{4}\frac{dV(t)}{dt} = \frac{1}{3}$$

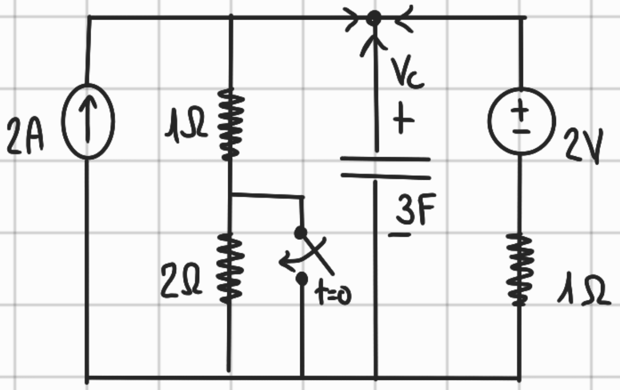
$dV(t)/dt = 4/3\ V$

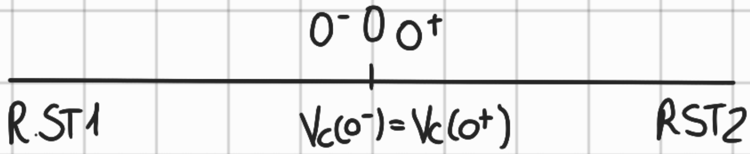
$$\Rightarrow \begin{cases} V(t) = K_1 e^{-t}\cos(\sqrt{3}t) + K_2 e^{-t}\operatorname{sen}(\sqrt{3}t) + 1 \\ \dfrac{dV(0^+)}{dt} = \dfrac{4}{3}\ V/s \\ V(0^+) = 1/3\ V \end{cases}$$

$K_1 = -2/3$

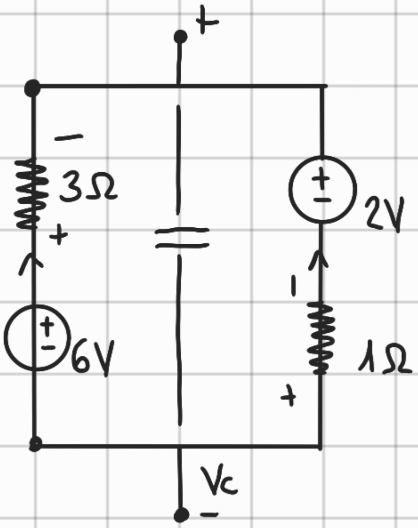
$K_2 = 2\sqrt{3}/9$

2 2006 es 3 Determina $V_c(t)$ risposta libera + forzata OK





• $t=0^-$ trasformo



Millman $V_c = \dfrac{2\cdot 1 + 6\cdot\frac{1}{3}}{1+\frac{1}{3}} = 3$

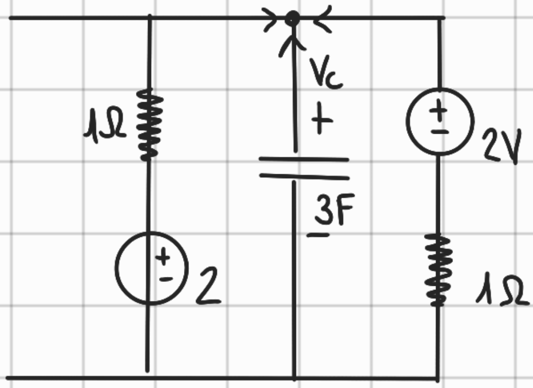
$V_c(0^-) = V_c(0^+) = 3\ V$

$V_P(t) = 3e^{-t/RC}$

$= 3e^{-t\,2/3}\ V$

$R(0^+)\quad 1\|1 = \dfrac{1}{1+1} = \dfrac{1}{2}$

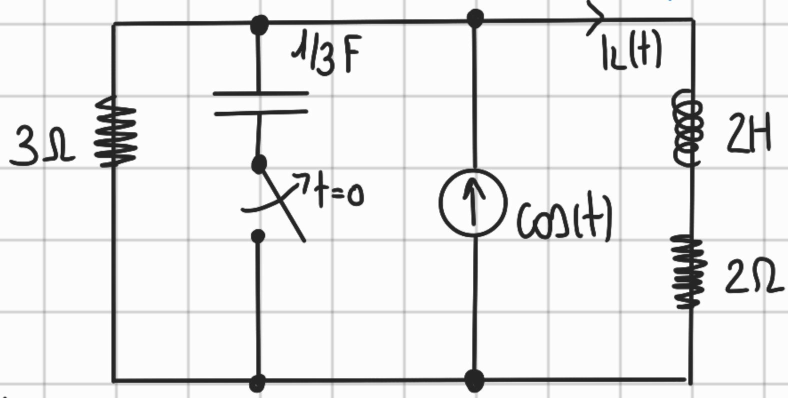
$RC = \dfrac{1}{2}\cdot 3 = \dfrac{3}{2}$



$V(\infty) = 2\ V$

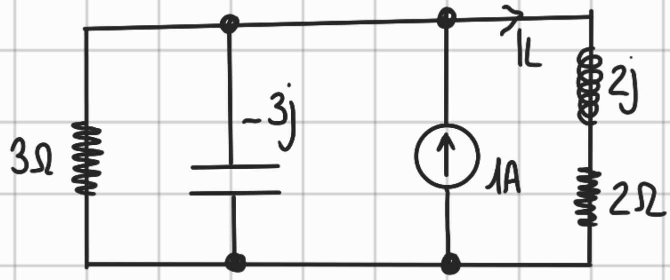
$V_{for} = 2(1 - e^{-t\,2/3})\ V$

14 DIC 2006 es3 OK Determina $I_L(t)$ per $t>0$ risposta libera e forzata



$t=0^-$

LIBERA $I_L(t) = I_0 e^{-t/\tau_{eq}}$ $\qquad \tau_{eq} = \frac{L}{R_{eq}}$

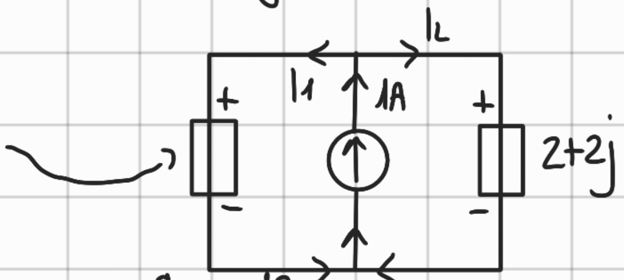


$I_L(0^-)? = I_L(0^+)$

$2+2j$

$\Downarrow$

$\| \quad \frac{-3j \cdot 3}{3-3j} = \frac{3}{2} - \frac{3}{2}j$



KI: $1 = I_1 + I_L \qquad I_L = 1 - I_1$

KV: $(2+2j)I_L - \left(\frac{3}{2} - \frac{3}{2}j\right) I_1 = 0$

$I_L = \frac{9}{25} - \frac{12}{25}j$

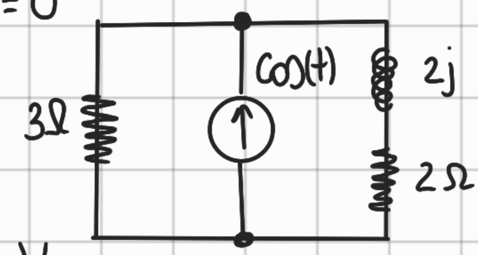
$I_L(0^-) = \frac{9}{25}\ A = I_L(0^+)$

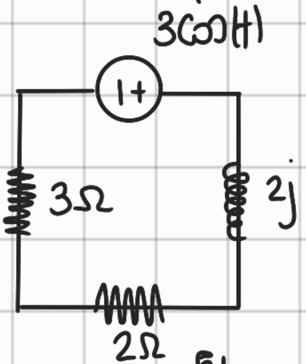
$R_{eq} \overset{Z}{=} 3+2 = 5\ \Omega$

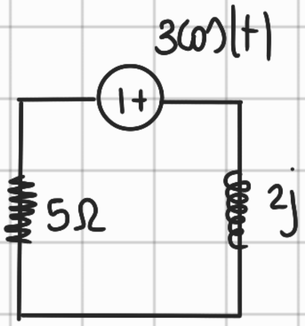
$I_{lib}(t) = \frac{9}{25} e^{-\frac{5}{2}t}\ A$

$\tau_{eq} = \frac{L}{R_{eq}} = \frac{2}{5}$

$t=0^+$







$V_M \cos(t) \qquad 2H \quad 5\Omega$

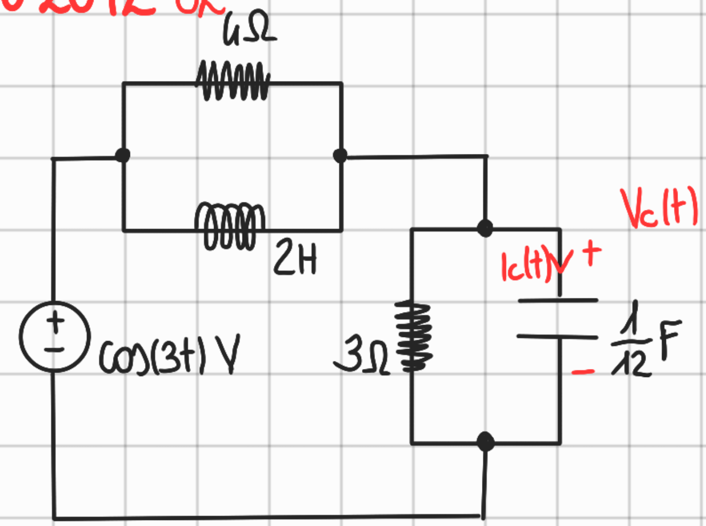
$\| $

3

$I_L(t) = -\frac{3}{\sqrt{5^2+2^2}} \cos\left(\arctan\left(\frac{2}{5}\right)\right) e^{-\frac{5}{2}t} + \frac{3}{\sqrt{5^2+2^2}} \cos\left(t - \arctan\left(\frac{2}{5}\right)\right) =$

$= -0{,}51 e^{-5/2\, t} + 0{,}557 \cos(t - 21{,}80°)$

7 GIU 2012 OK



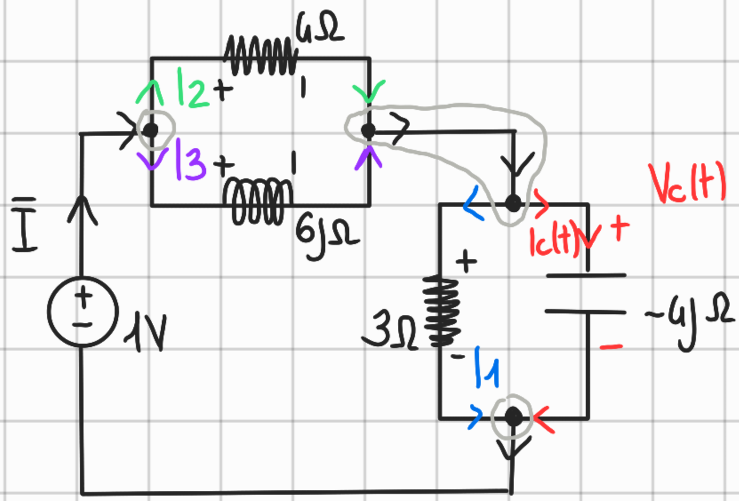
Determina Vc(t) e Ic(t) in regime permanente sinusoidale

Porto tutto nel dominio dei fasori $\omega = 3$ rad/s

$Z_L = j\omega L = 6j\,\Omega$

$Z_C = \frac{1}{j\omega C} = -4j\,\Omega$

$\bar{V}_g = e^{j0} = 1\,V$

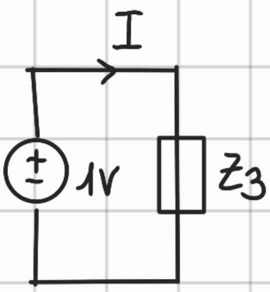


Trovo I semplificando

$$Z_1 = 4 \parallel 6j = \frac{4 \cdot 6j}{4+6j} = \frac{24j}{4+6j} \cdot \frac{4-6j}{4-6j} = \frac{96j+144}{16+36} = \frac{96j+144}{52}$$

$$= \frac{72+48j}{26} = \frac{36+24j}{13}\,\Omega$$

$$Z_2 = 3 \parallel (-4j) = \frac{3(-4j)}{3-4j} = \frac{48-36j}{25}\,\Omega$$



$$Z_3 = Z_1 + Z_2 = \frac{36+24j}{13} + \frac{48-36j}{25} = \frac{1524}{325} + \frac{132i}{325}$$

$$\bar{V} = Z_3 \bar{I} \qquad \bar{I} = \frac{\bar{V}}{Z_3} = \frac{127}{600} - \frac{11j}{600}$$

Trovo tutte le altre correnti con KV e KI

KI: $\begin{cases} I_2+I_3 = I_1+I_C \\ I_1+I_C = I \\ I_2+I_3 = I \end{cases}$ KV: $\begin{cases} -V_L+V_{R2}=0 \\ V_C - V_{R1} = 0 \\ V_{R1} - 1 + V_L = 0 \end{cases} \rightsquigarrow \begin{cases} -6jI_3+4I_2=0 \\ V_C = 3I_1 \\ 3I_1 - 1 + 6jI_3 = 0 \end{cases}$

$V_C = 3I_1$ basta trovare I_1 $\quad 3I_1 = 1-6jI_3$

$I_C = I - I_1$

• $I_1 = \frac{1}{3} - 2jI_3$ • $-3jI_3+2I_2=0$

$\rightsquigarrow 3jI_3 = 2I_2 \quad I_2 = I - I_3 \quad 3jI_3 = 2I - 2I_3 \quad (3j+2)I_3 = 2I$

$I_3 = \frac{2I}{2+3j} \Rightarrow I_1 = \frac{1}{3} - \frac{4jI}{2+3j}$

Quindi $V_C = 1 - \frac{12jI}{2+3j} = \frac{19}{50} - \frac{17}{50}j$ V

$I_C = I - \left(\frac{1}{3} - \frac{4jI}{2+3j}\right) = \frac{17}{200} + \frac{19}{200}j$ A

$V_C(t) = \sqrt{\left(\frac{19}{50}\right)^2+\left(\frac{17}{50}\right)^2}\cos\left(3t - \arctan\left(\frac{17}{19}\right)\right) = 0{,}51\cos(3t-42°)$ V

$I_C(t) = \sqrt{\left(\frac{17}{200}\right)^2+\left(\frac{19}{200}\right)^2}\cos\left(3t + \arctan\left(\frac{19}{17}\right)\right) = 0{,}127\cos(3t+48°)$ A

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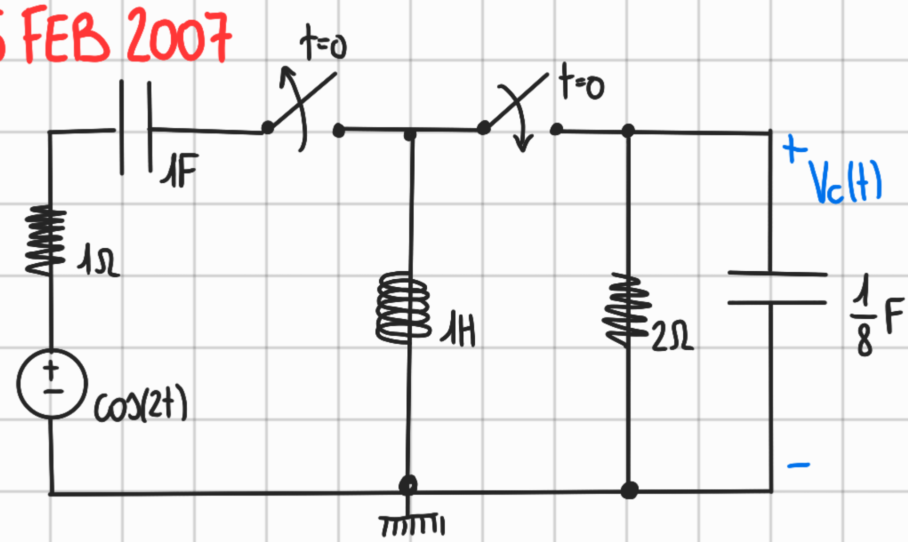
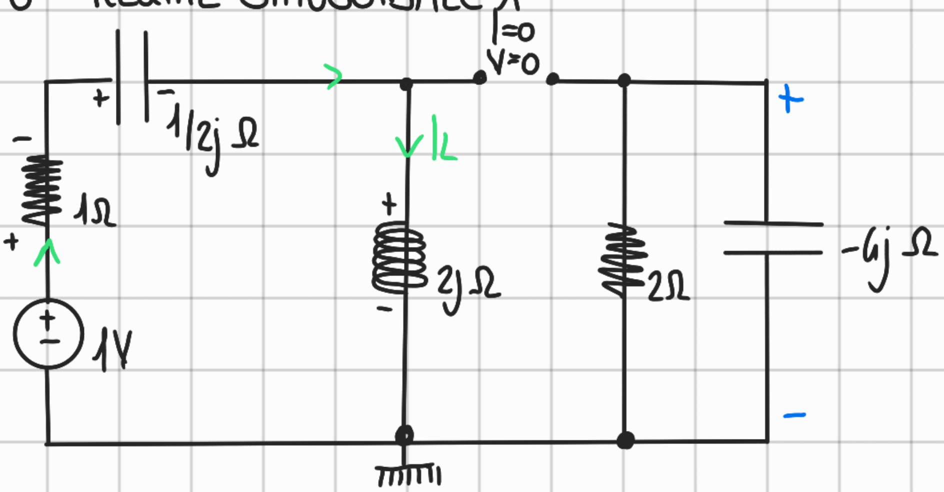


Determina Vc(t)

	0^-	0	0^+
REG SIN I			$V_C(0^-)=V_C(0^+)$ $I_L(0^-)=I_L(0^+)$

- $t=0^-$ REGIME SINUSOIDALE $\omega=2$

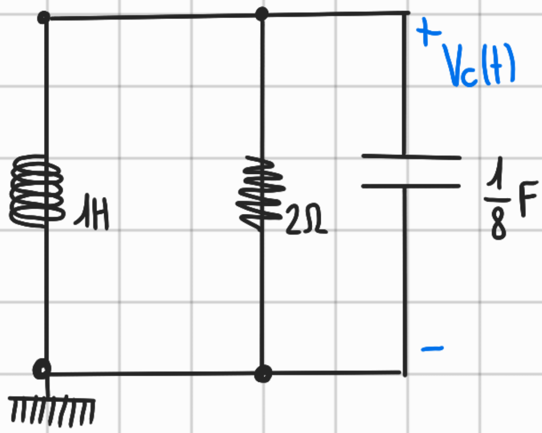


KV: $+2jI_L-1+1I_L+\frac{1}{2j}I_L=0 \rightsquigarrow -4I_L-2j+2jI_L+I_L=0$

$(2j-3)I_L=2j \qquad I_L=\frac{2j}{2j-3}=\frac{4}{13}-\frac{6}{13}j$

- $I_L(0^-)=I_L(0^+)=4/13$ A
- $V_C(0^-)=0$!!!

- $t=\infty$



RLC parallelo

$\omega_0=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{1/8}}=2\sqrt{2}\sim 2{,}82$

$\alpha_p=\frac{G_p}{2C}=\frac{1}{2\cdot 2\frac{1}{8}}=2$

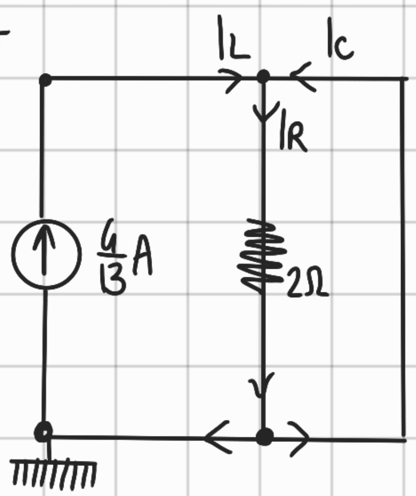
$\alpha<\omega_0$

SOTTOSMORZATO

$$S_{1,2} = -2 \pm \sqrt{(2\sqrt{2})^2 - (2)^2}\, j = -2 \pm 2j \qquad \begin{matrix} Re = -2 \\ Im = 2 \end{matrix}$$

$$V_c(t) = K_1 e^{-2t}\cos(2t) + K_2 e^{-2t}\operatorname{sen}(2t) + \underset{0}{\underset{\|}{\cancel{R2}}}$$

• $t = 0^+$



$V_c(0) = 0$

$I_L(0) = 4/13\,A$

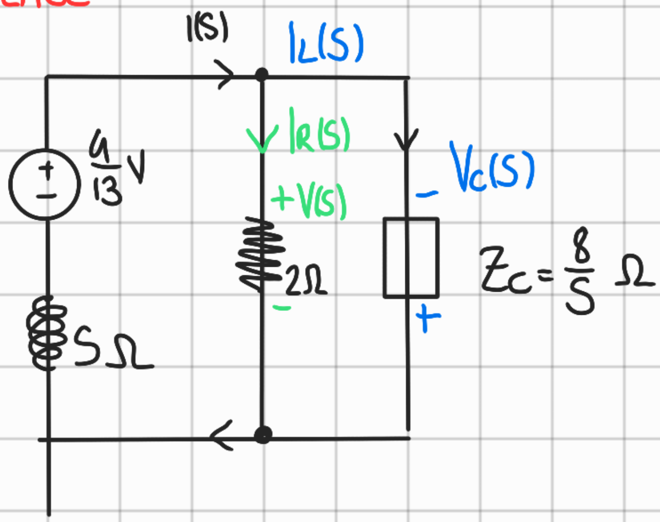
$I_L + I_c = I_R$

$$\frac{4}{13} + C\frac{dV(t)}{dt} = 0 \qquad \frac{4}{13} - \frac{1}{8}\frac{dV(t)}{dt} = 0$$

$dV(t)/dt = 32/13\ V/s$

$$\Rightarrow \begin{cases} V_c(t) = K_1 e^{-2t}\cos(2t) + K_2 e^{-2t}\operatorname{sen}(2t) \\ V_c(0) = 0\ V \\ \dfrac{dV_c(t)}{dt} = -32/13\ V/s \end{cases} \Rightarrow V_c(t) = -\frac{16}{13} e^{-2t}\sin(2t)$$

LAPLACE



$$V_c(S) = \frac{\frac{4}{13}\frac{1}{S}}{\frac{1}{S} + \frac{1}{2} + \frac{S}{8}} = \frac{32}{13}\frac{1}{S^2 + 4S + 8} = \frac{2^5}{13\left((S+2)^2 + 2^2\right)} = \frac{8}{13}\frac{4}{(S+2)^2 + 4}$$

$$\mathcal{L}^{-1}\{V_c(S)\} = \frac{8}{13} e^{-2t}\operatorname{sen}(2t)$$