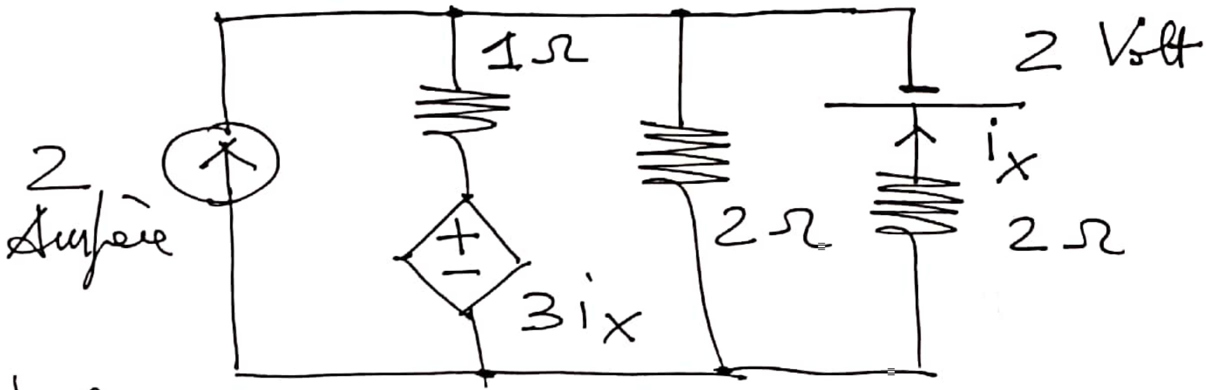
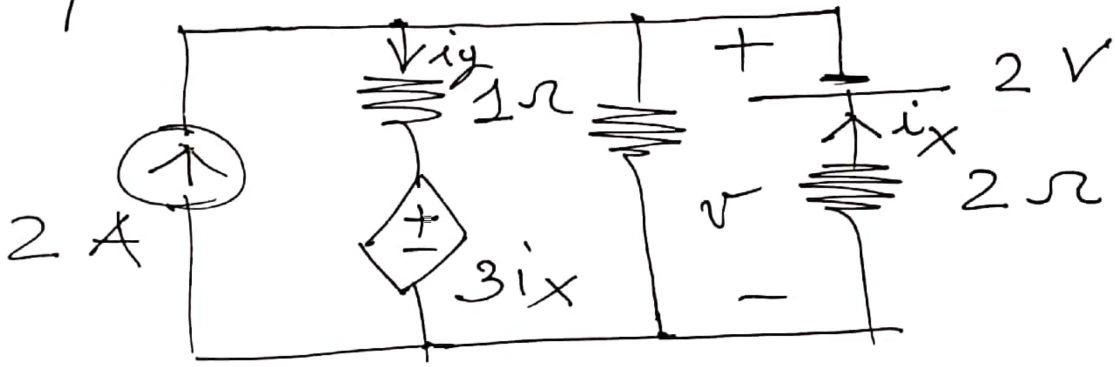
(1)

**Esempio** Verificare il teorema di Tellegen per il circuito in figura.




**Soluzione.**




Si ha:

$$v = \frac{2 + 3i_x - \frac{2}{2}}{1 + \frac{1}{2} + \frac{1}{2}} = \frac{1}{2} + \frac{3}{2} i_x$$

$$v = -2 - 2i_x \Rightarrow i_x = -1 - \frac{v}{2}$$

da cui

$$v = -\frac{4}{7}\,V\,; \quad i_x = -\frac{5}{7}\,A.$$

Inoltre:

$$i_x = -1 - \frac{1}{2}\left(-\frac{4}{7}\right) = -\frac{5}{7}\,A$$

$$v = i_y + 3i_x \Rightarrow i_y = v - 3i_x = -\frac{4}{7} + \frac{15}{7} = \frac{11}{7}\,A.$$

Potenze assorbite dai bipoli:

$$P_{\substack{\text{gen.re}\\ \text{corrente}}} = -2 \cdot v = \frac{8}{7}\,W$$

$$P_{1\Omega} = 1 \cdot i_y^2 = \frac{121}{49}\,W$$

$$P_{\substack{\text{gen.re}\\ \text{controllato}}} = 3 i_x \cdot i_y = 3\left(-\frac{5}{7}\right)\left(\frac{11}{7}\right) = -\frac{165}{49}\,W$$

$$P_{\substack{2\Omega\\ sx}} = \frac{v^2}{2} = \frac{1}{2}\,\frac{16}{49} = \frac{8}{49}\,W$$

$$P_{\text{batteria}} = 2 \cdot i_x = -\frac{10}{7}\,W$$

$$P_{\substack{2\Omega\\ dx}} = 2 \cdot i_x^2 = 2 \cdot \frac{25}{49} = \frac{50}{49}\,W$$

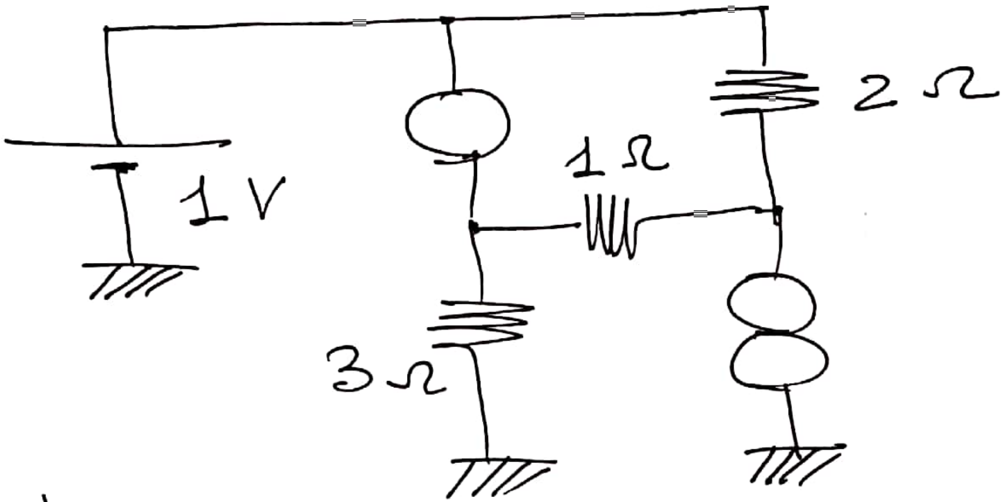
Si ha:

$$\frac{8}{7} + \frac{121}{49} - \frac{165}{49} + \frac{8}{49} - \frac{10}{7} + \frac{50}{49} =$$

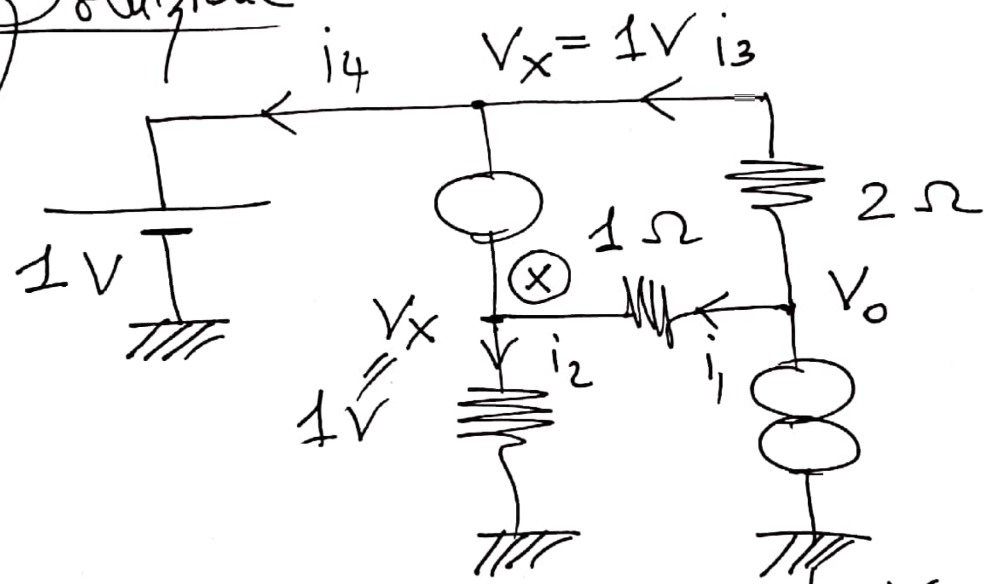
$$= \frac{56 + 121 - 165 + 8 - 70 + 50}{49} = 0.$$

Tellegen è verificato.

**Esempio** Verificare il teorema di Tellegen per il circuito in figura.




**Soluzione**

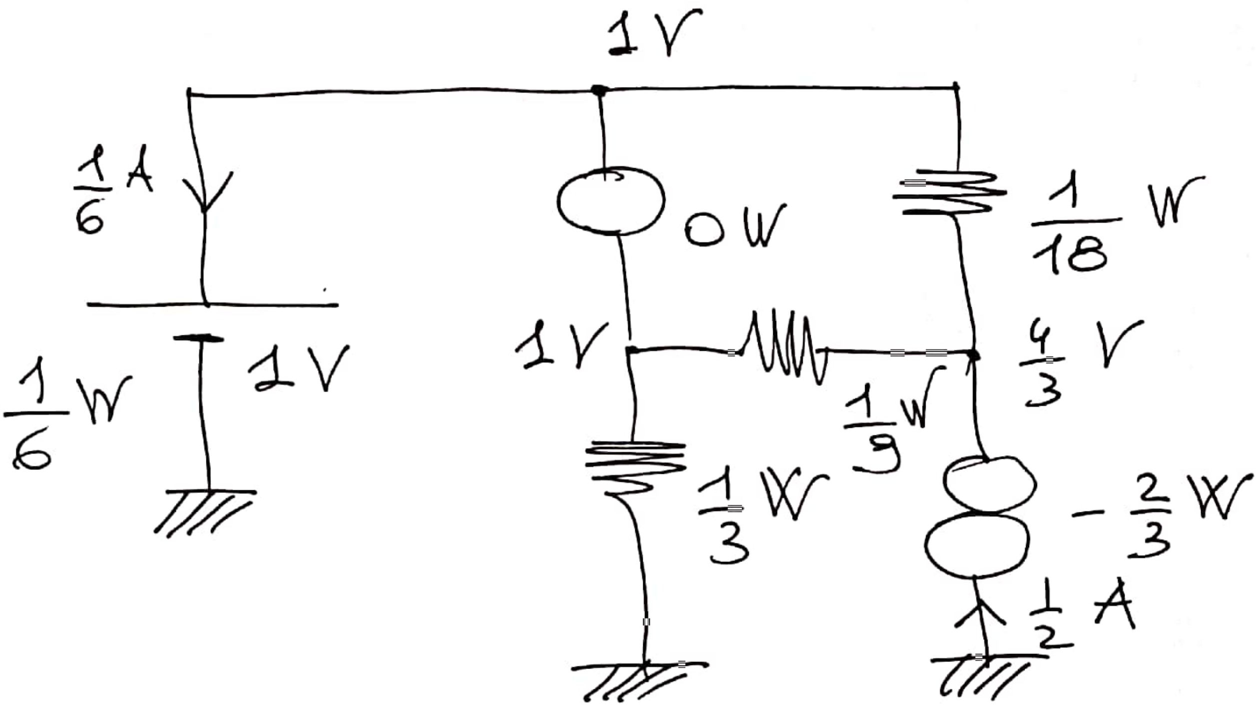



$V_X = 1\,V$; unica incognita $V_0$

KI nodo ⊗: $\dfrac{V_0 - 1}{1} = \dfrac{1}{3} \Rightarrow \boxed{V_0 = \dfrac{4}{3}\,V}$

$\Rightarrow i_1 = i_2 = \dfrac{1}{3}\,A$; $i_3 = i_4 = \dfrac{1}{6}\,A$.

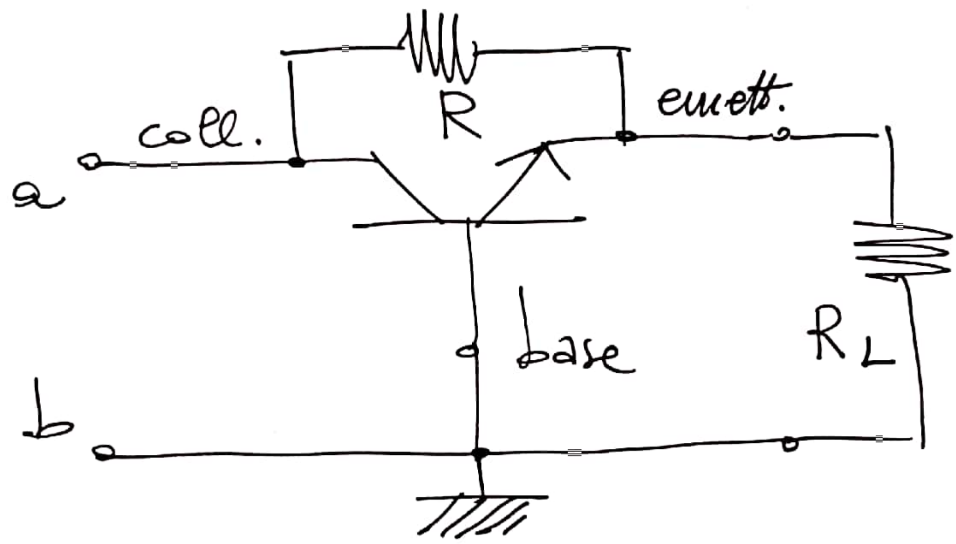
Ne segue la situazione delle potenze assorbite come in figura.



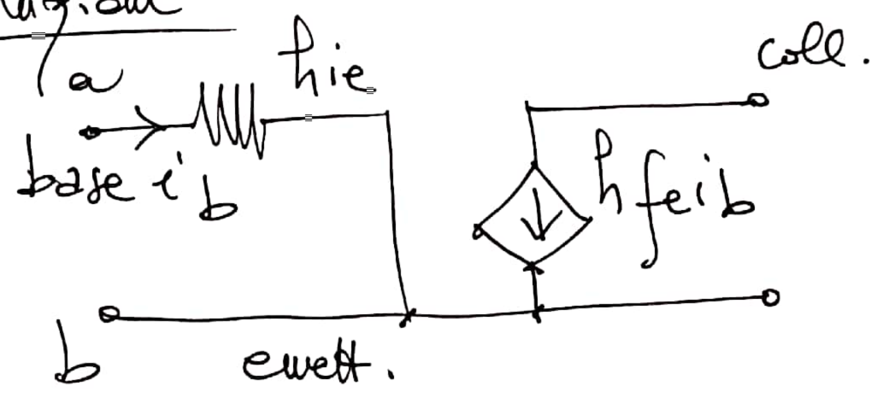

$$\frac{1}{6}+\frac{1}{3}+\frac{1}{9}+\frac{1}{18}=\frac{12}{18}=\frac{2}{3}\ W.$$

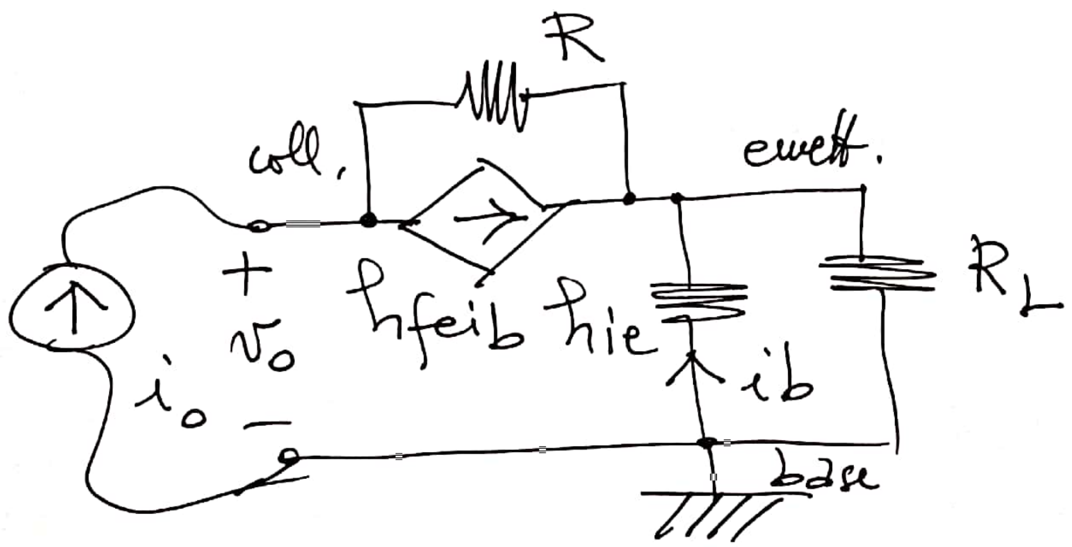
È verificato il teorema di Tellegen.

Esempio. ? Req bipolo ab usando per il transistor il modello a parametri h a emettitore comune con $h_{re} = 0$ e $h_{oe} = 0$ Ohm$^{-1}$. (1)

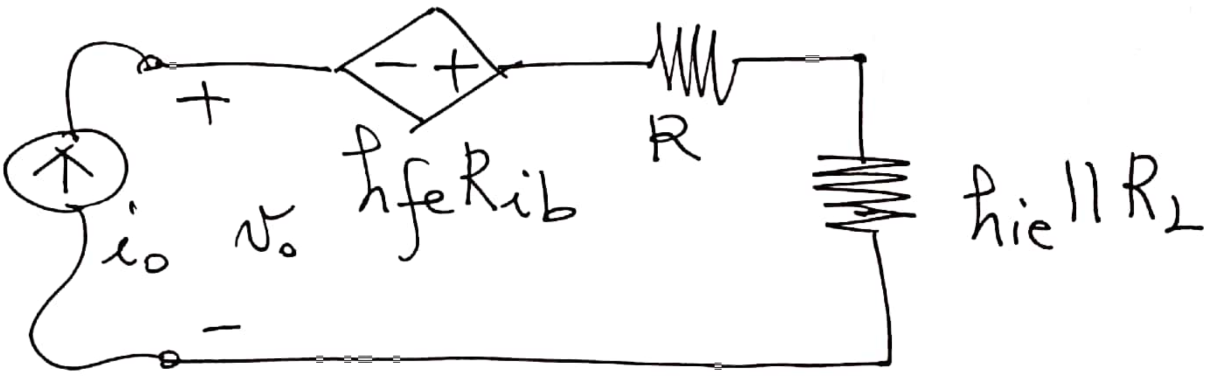



Solu



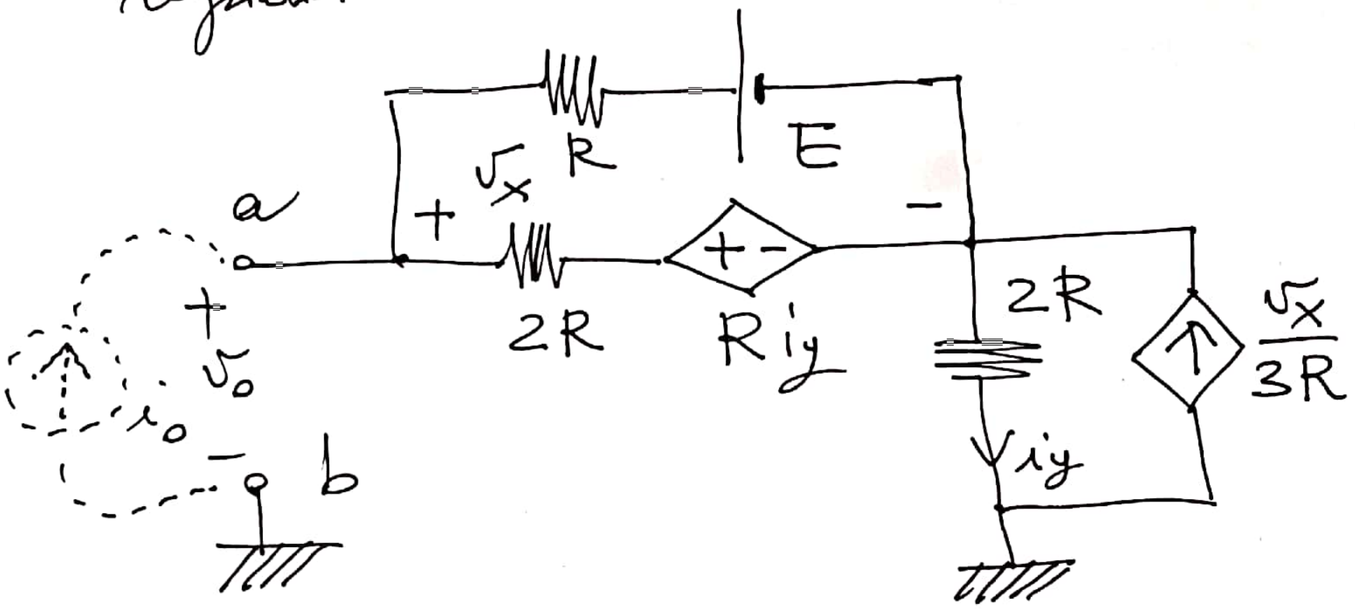
$$i_b = -i_o \frac{\frac{1}{h_{ie}}}{\frac{1}{h_{ie}} + \frac{1}{R_L}} = -i_o \frac{R_L}{R_L + h_{ie}}$$




$$v_o = -h_{fe}Ri_b + Ri_o + h_{ie} \| R_L i_o$$
$$= h_{fe}\frac{R R_L}{R_L + h_{ie}} i_o + Ri_o + h_{ie} \| R_L i_o$$

$$\Rightarrow R_{eq} = \frac{v_o}{i_o} = h_{fe}\frac{R R_L}{R_L + h_{ie}} + R + h_{ie} \| R_L .$$

Esempio Determinare l'equivalente di Thevenin del bipolo ab usando il metodo rapido.




Soluzione Si ha

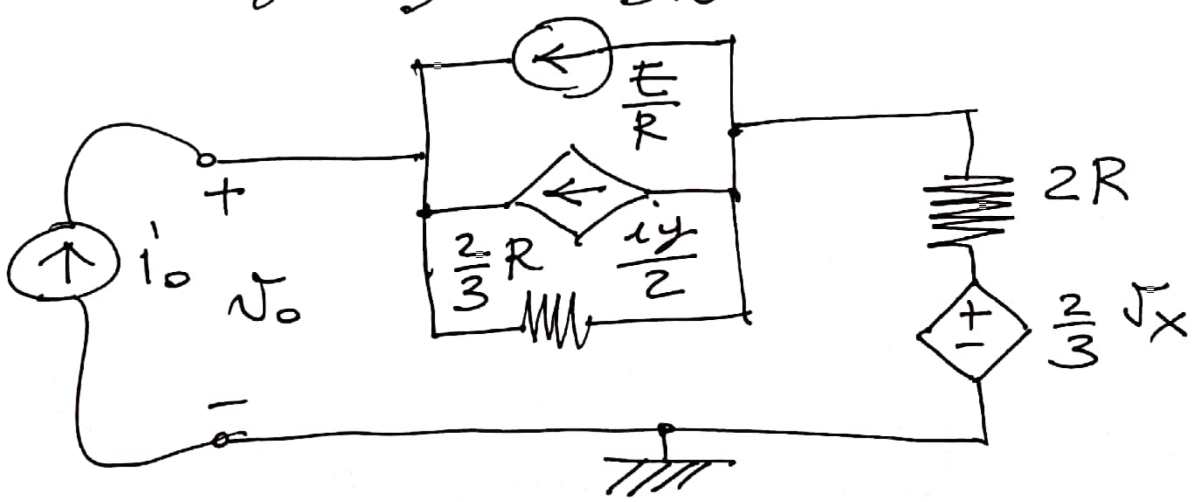
$$i_o = i_y - \frac{v_x}{R}$$

$$v_o = v_x + 2Ri_y$$

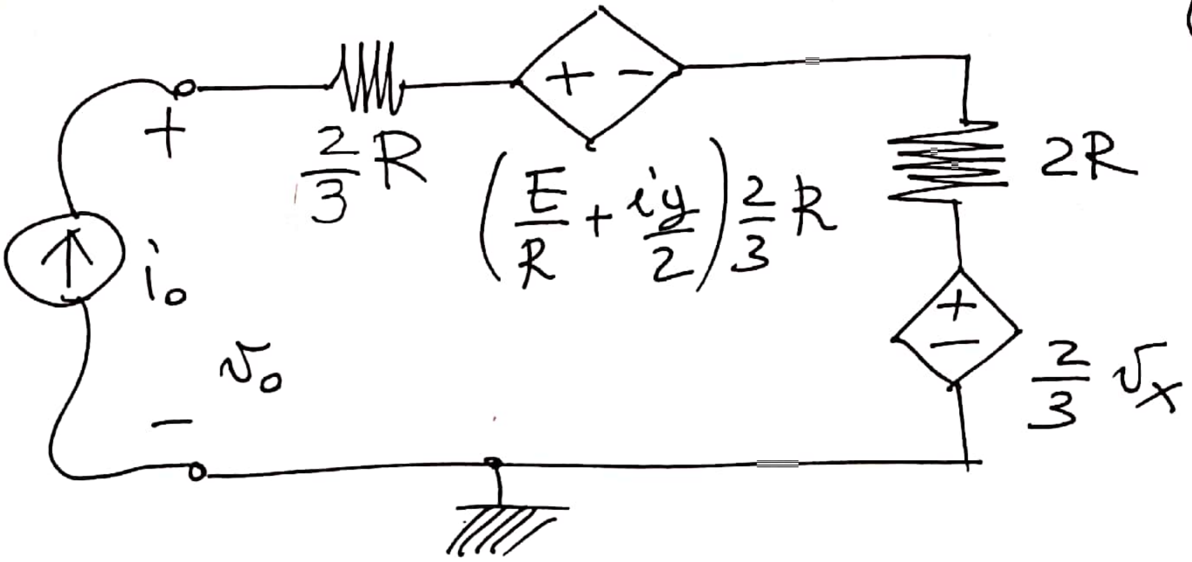
Risolvendo:

$$v_x = \frac{3}{5}v_o - \frac{6}{5}Ri_o$$

$$i_y = \frac{3}{5}i_o + \frac{v_o}{5R}$$



2



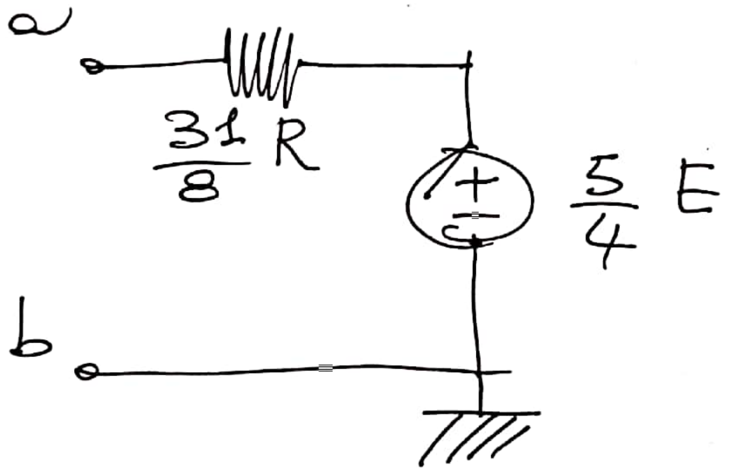

$$v_0 = \frac{8}{3}Ri_0 + \frac{2}{3}E + \frac{Ri_y}{3} + \frac{2}{3}v_x$$

$$= \frac{8}{3}Ri_0 + \frac{2}{3}E + \frac{Ri_0}{5} + \frac{v_0}{15} + \frac{2}{5}v_0 - \frac{4}{5}Ri_0$$

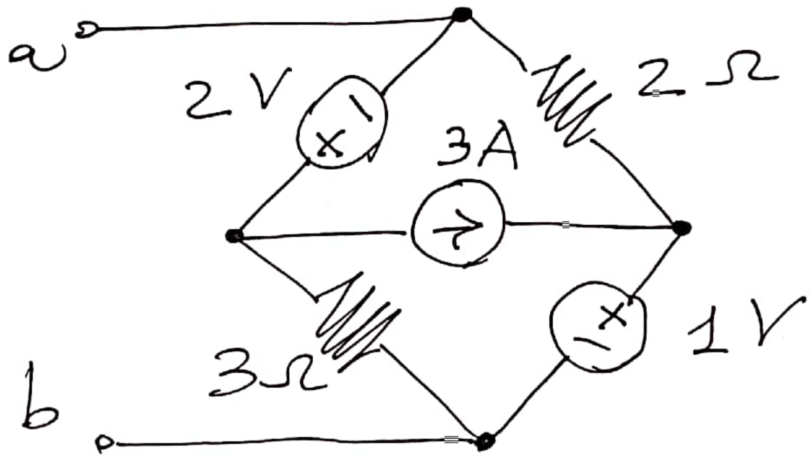
Riarrangiando:

$$v_0 = \frac{31}{8}Ri_0 + \frac{5}{4}E.$$

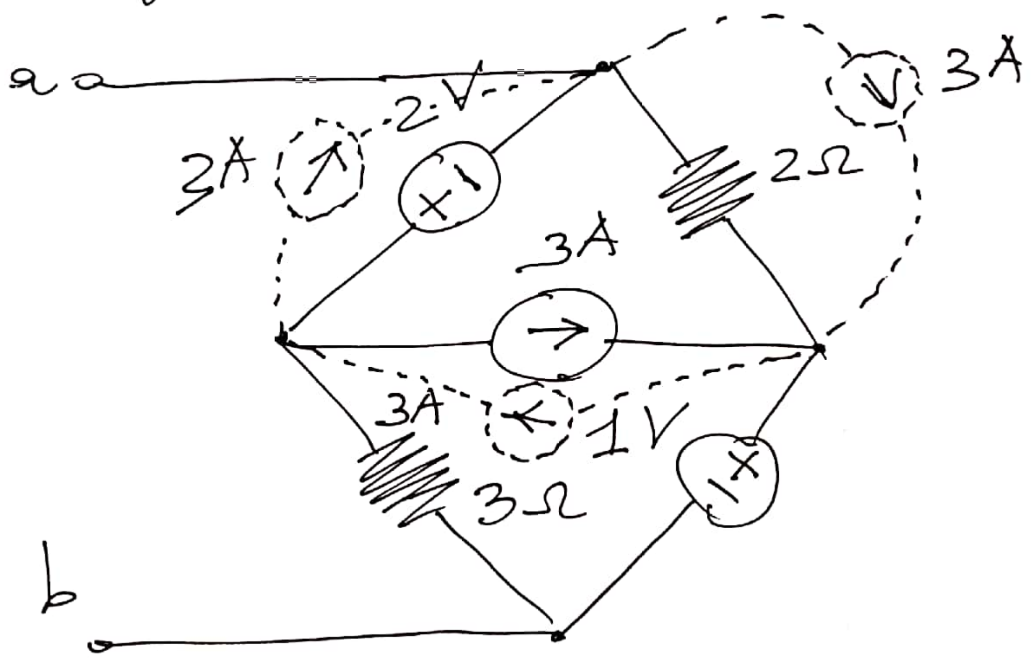
Quindi:

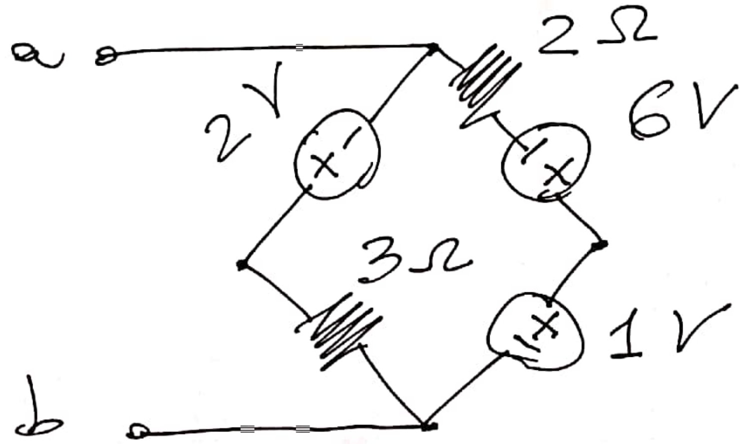


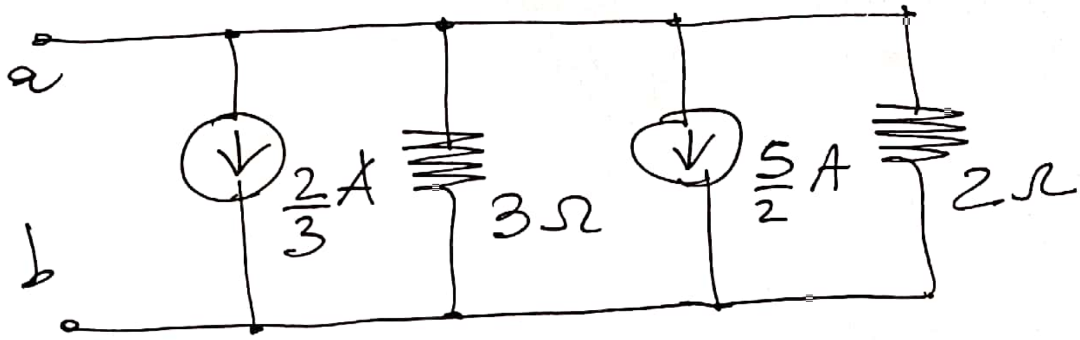
Esempio: ? Norton bipolo ab usando trasformazioni circuitali.

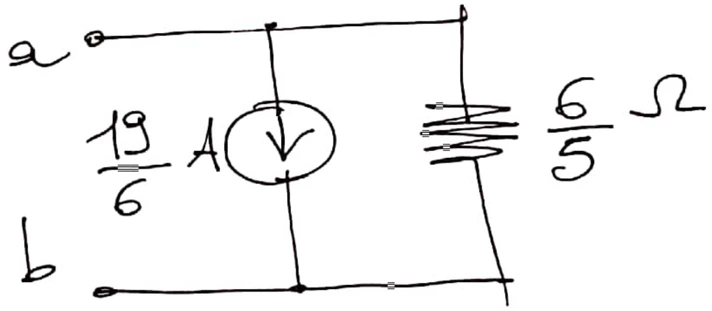



Soluzione:

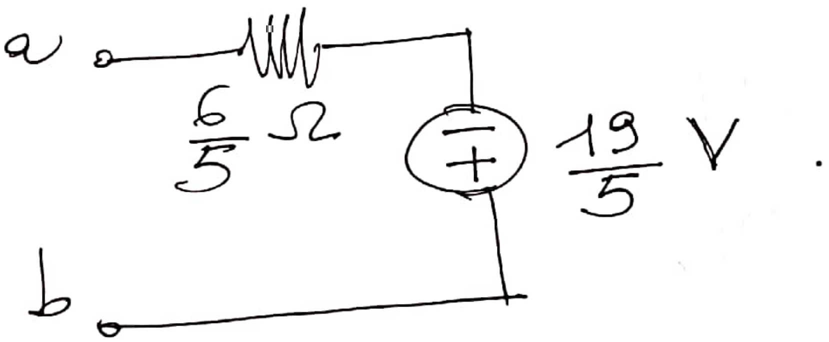



a
b
$\frac{2}{3}$ A
3 Ω
$\frac{5}{2}$ A
2 Ω


a
b
$\frac{19}{6}$ A
$\frac{6}{5}$ Ω

Norton


a
b
$\frac{6}{5}$ Ω
$\frac{19}{5}$ V