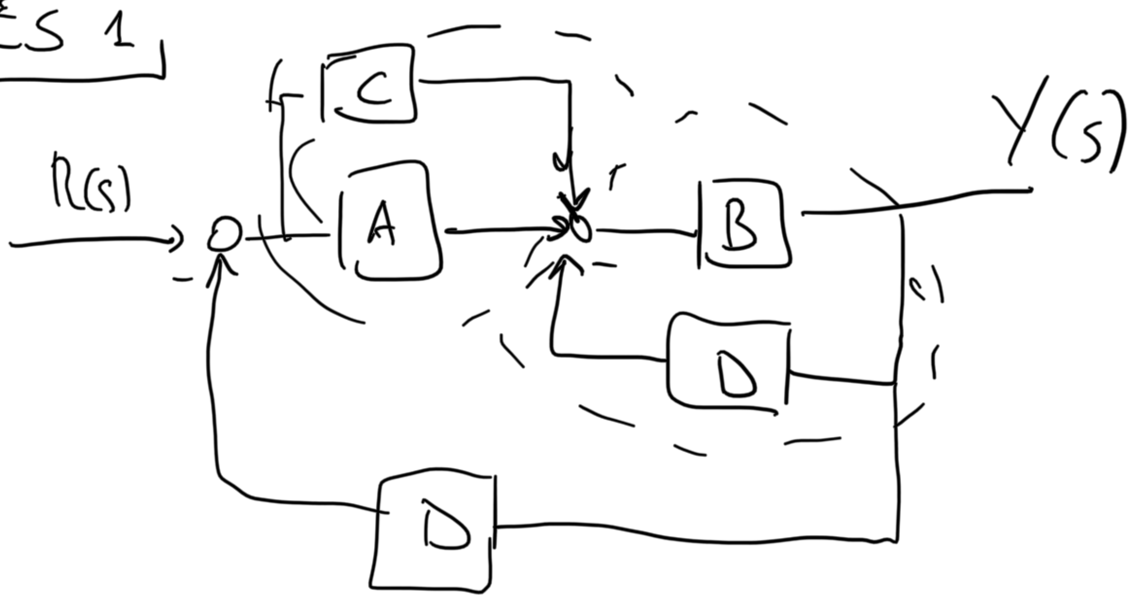
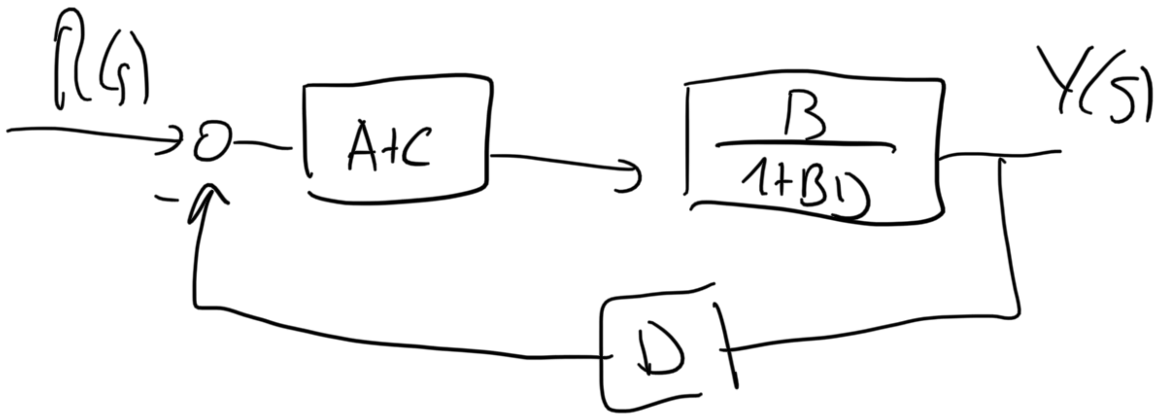
Soluzione Compitino 22/11/2021

FILA A

ES 1





$$G(s) = \frac{(A+C)B}{1+BD} \cdot \frac{1+BD}{1+BD+(A+C)BD} = \frac{AB+BC}{1+BD+ABD+BCD}$$

$A = \frac{3}{s}$, $B = \frac{1}{s+2}$, $C = 2$, $D = 2$

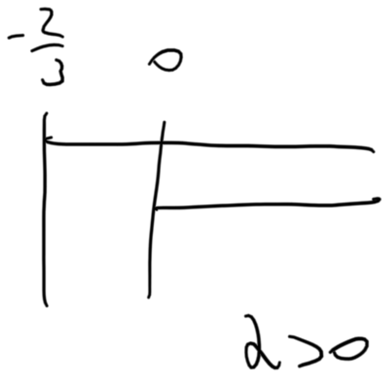
$$G(s) = \frac{3+2s}{s^2+2s+2s+3\,2+2\,2s} =$$

$3+2s$

$$= \frac{\dots}{s^2+(2+3\alpha)s+3\alpha}$$

Per Cartesio è stabile se

$$\begin{cases} 2+3\alpha > 0 \\ 3\alpha > 0 \end{cases}$$



ESERCIZIO 2

$$G(s) = \frac{-100\,(10s+1)}{(s+10)^2\,(s^2+10)}$$

FORMA DI BODE

$$G(s) = \frac{-100}{1000}\,\frac{(10s+1)}{\left(\frac{s}{10}+1\right)^2\left(\frac{s^2}{10}+1\right)}$$

$$K_B = -0.1 = -20\,dB$$

DIAGRAMMA BODE

$\uparrow |G(j\omega)|$

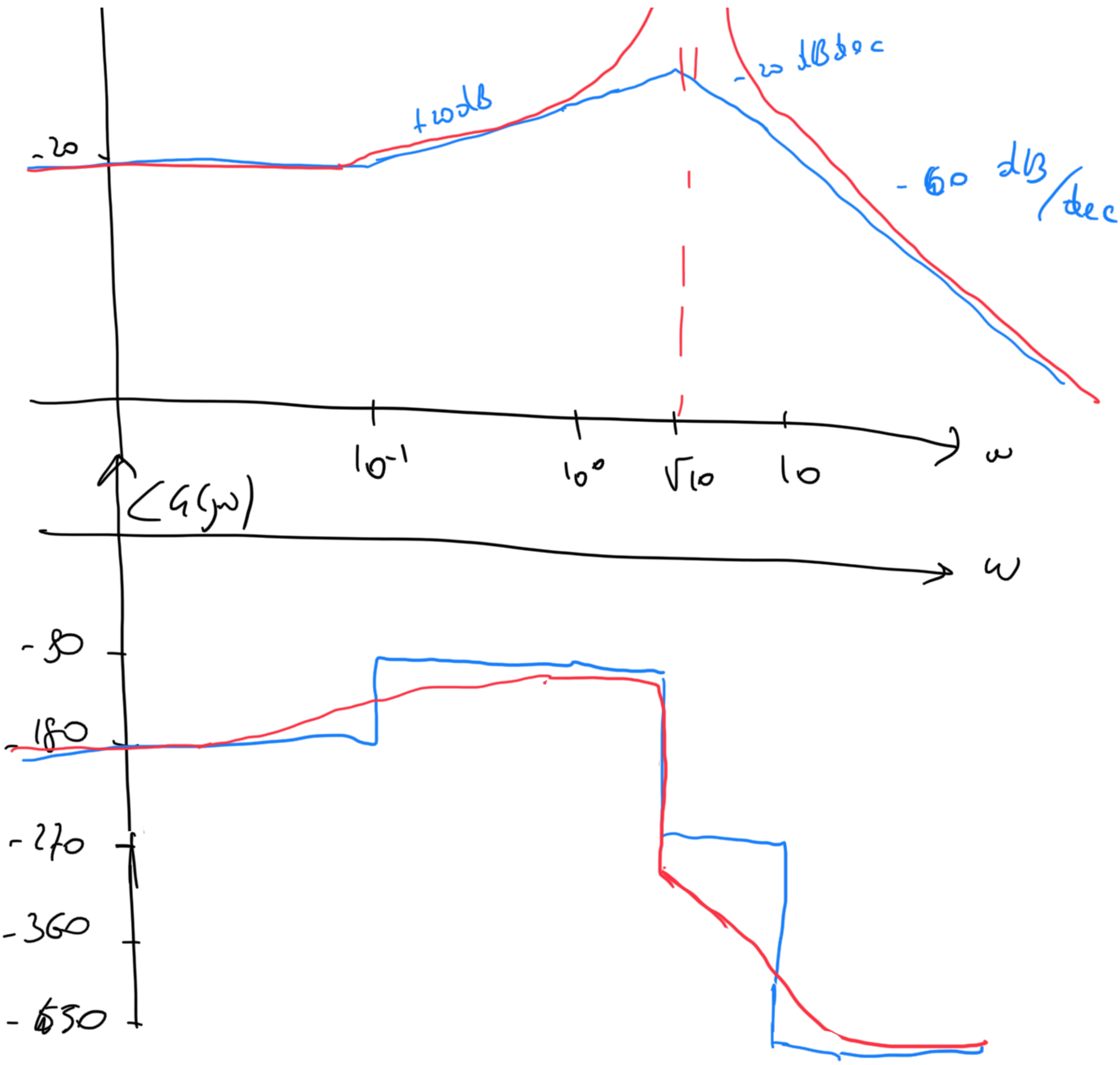
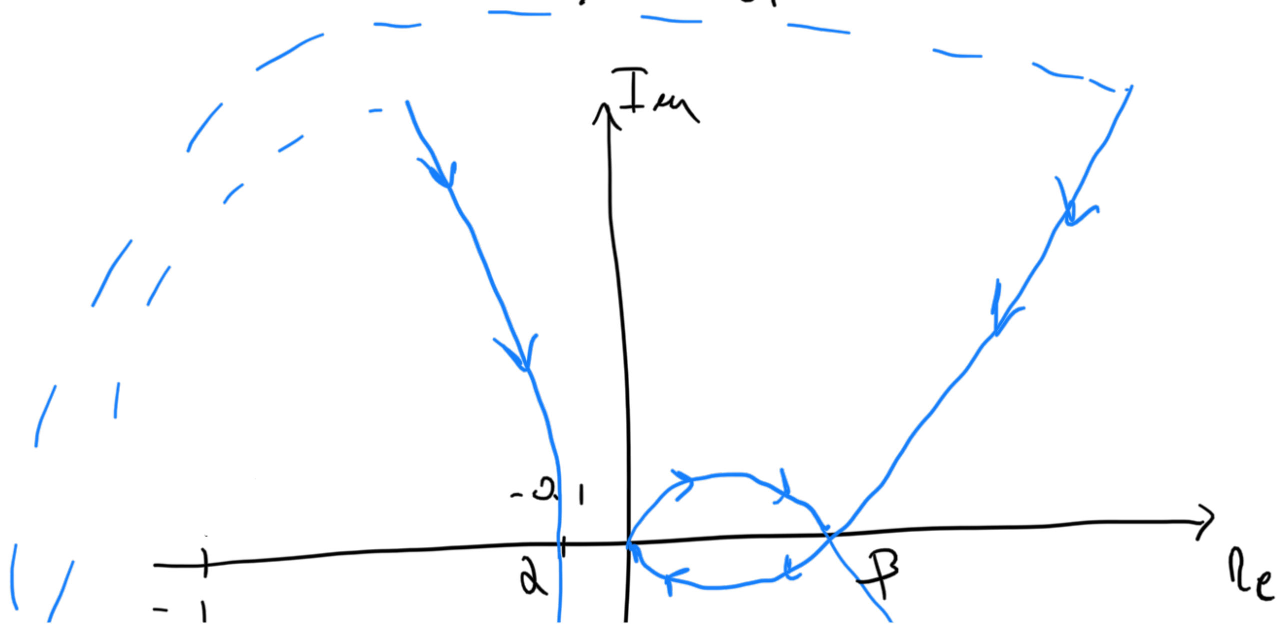
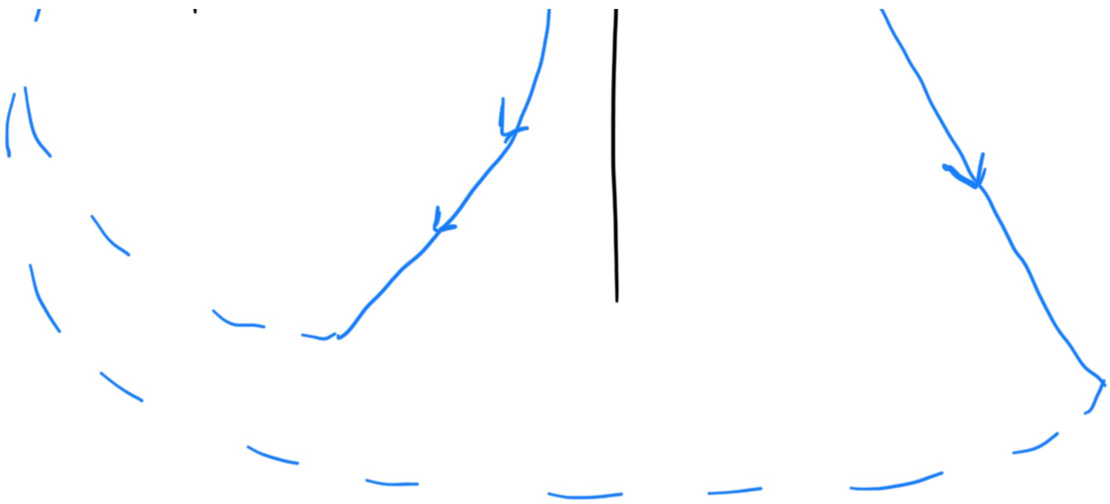


DIAGRAMMA DI NYQUIST





STABILITA'

$k>0$

$k > \frac{1}{\alpha} \to$ 1PD

$k < \frac{1}{\alpha} \to$ 2PD

$k<0$

$|k| > \frac{1}{\beta}$ $\quad k < -\frac{1}{\beta} \to$ 2PD

$|k| < \frac{1}{\beta} \to k > -\frac{1}{\beta}$ STABILE

ESERCIZIO 3

$$G(s) = \frac{100\,(-s-1)}{(s+16)^2\,(s^2+16)}$$

POLI

$s=-16 \quad \mu=2$

$s = \pm 4j$

ZERI

$s=-1$

FORMA POLI-ZERI

$$G(s) = \frac{-100\ (s+1)}{(s+16)^2\ (s-4j)(s+4j)}$$

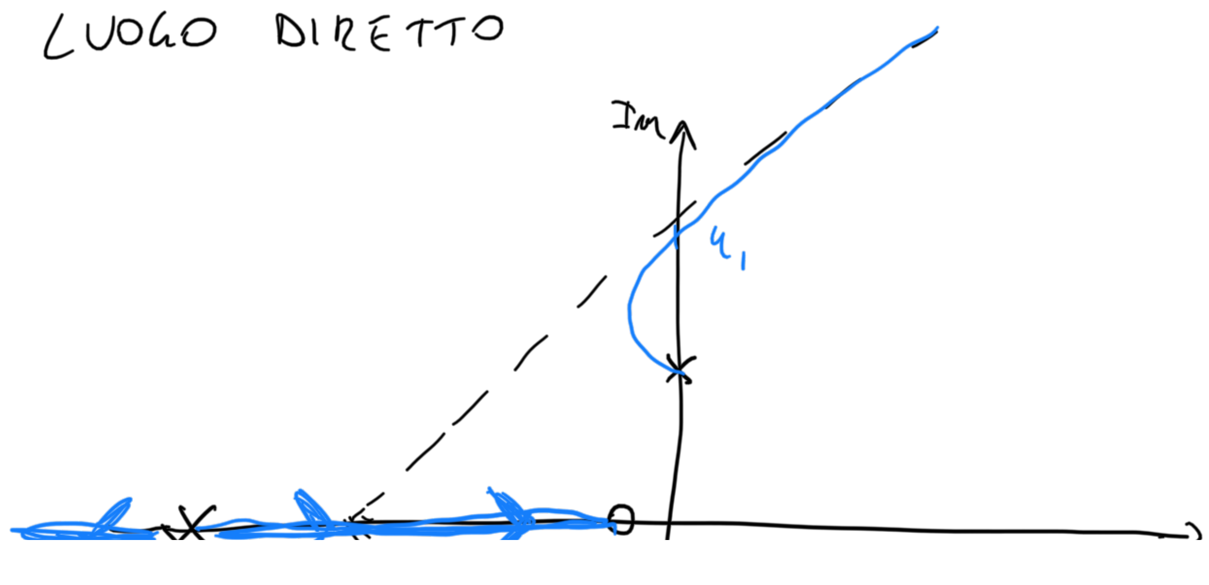
N.B. $k = -100$, nello studio della stabilità si considera il luogo complementare per $k>0$ e quello diretto per $k<0$

$$s_0 = \frac{-16-16+4j-4j+1}{n-m} = \frac{-31}{3} \simeq -10.33$$

$$\varphi_0 \quad \begin{cases} \pi/3\ ,\ \pi\ ,\ \frac{5}{3}\pi \\ 0\ ,\ \frac{2}{3}\pi\ ,\ \frac{4}{3}\pi \end{cases}$$

$\mu = 0,1,2$

LUOGO DIRETTO



CONDIZIONE DI FASE per $(s-4j)$

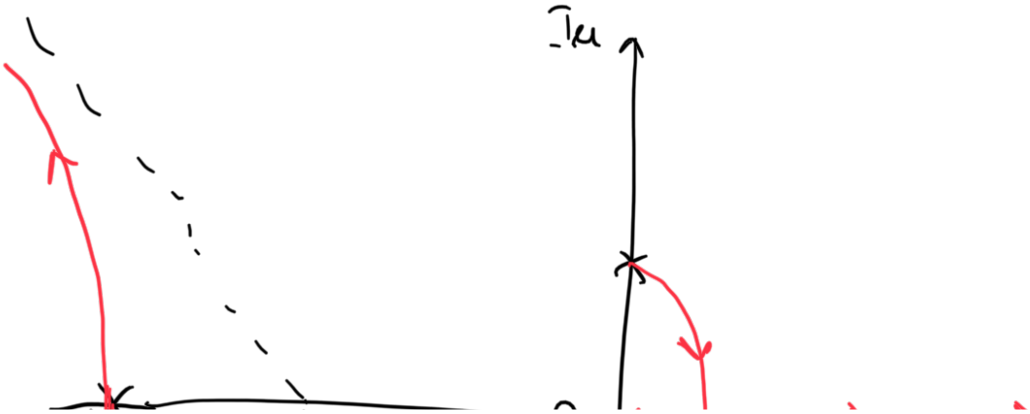
$$2\angle s+16 + \angle s-4j + \angle s+4j - \angle s+1 = \pi$$

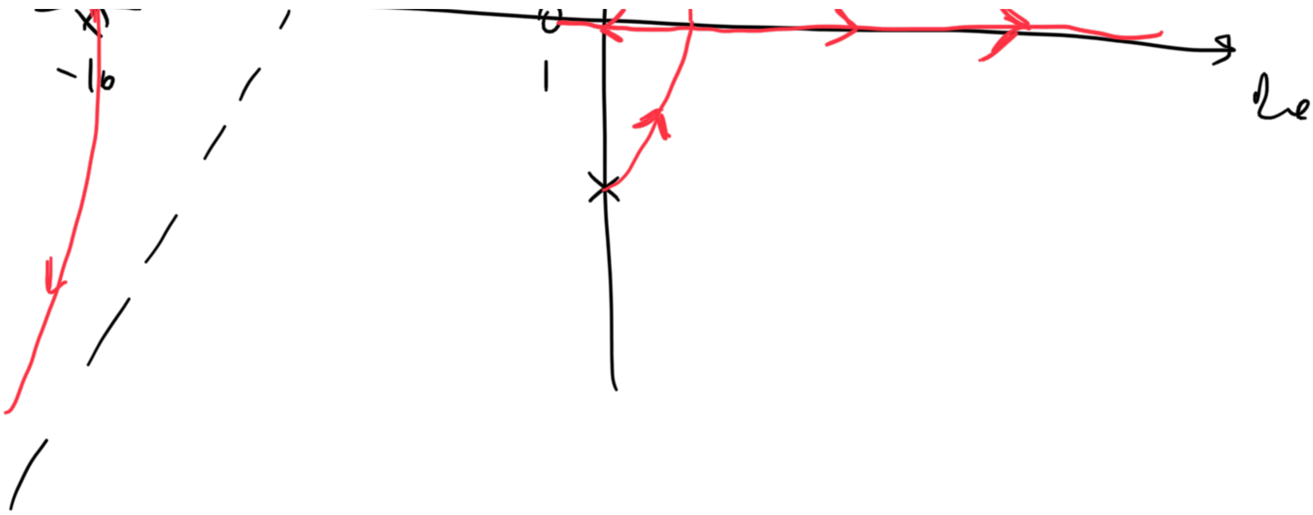
$$2\angle 4j+16 + X + 8j - \angle 4j+1 = \pi$$

$$2 \cdot 14.03 + X + 90 - 75{,}96 = 180$$

$$X = 137{,}9 \rightarrow \text{II quadrante}$$

LUOGO COMPLEMENTARE





Condizione di fase per $(s-4j)$

$$x = -42.1 \rightarrow \text{IV quadrante}$$

STABILITA'

$k < 0$ (LD)

$|k| > k_1 \rightarrow 2\,PD$

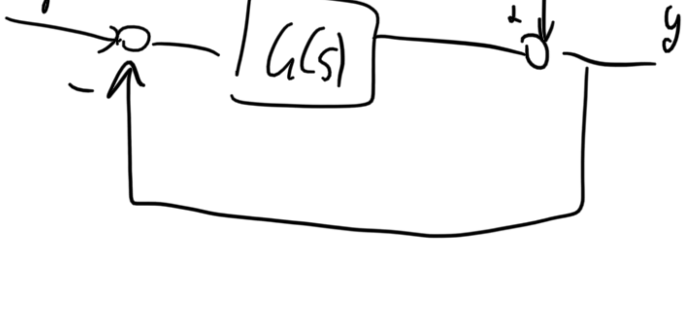
$|k| < k_1$ STABILE

$k > 0$ (LC)

$k > K_2 \rightarrow 1\,PD$

$k < k_2 \rightarrow 2\,PD$

Esercizio 4



$$T(s) = \frac{D(s)}{Y(s)} = \frac{1}{1+G} = \frac{1}{1+\frac{3}{s}} = \frac{s}{s+3}$$

$$Y(s) = D(s) \cdot T(s) = \frac{6}{s^2} \cdot \frac{s}{s+3}$$

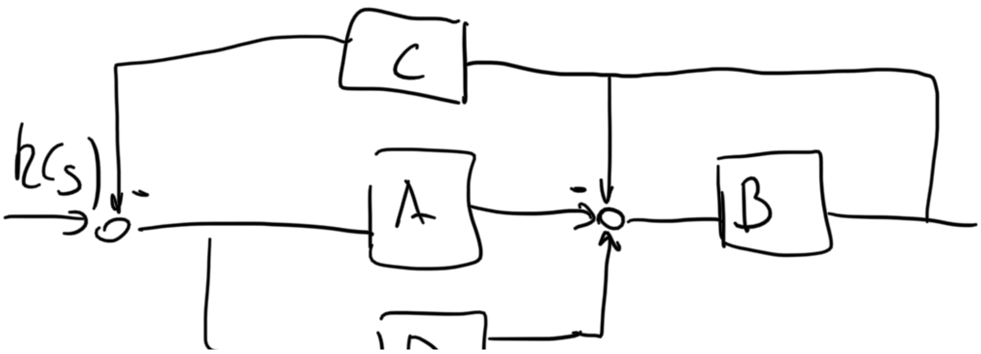
con Th valore finale

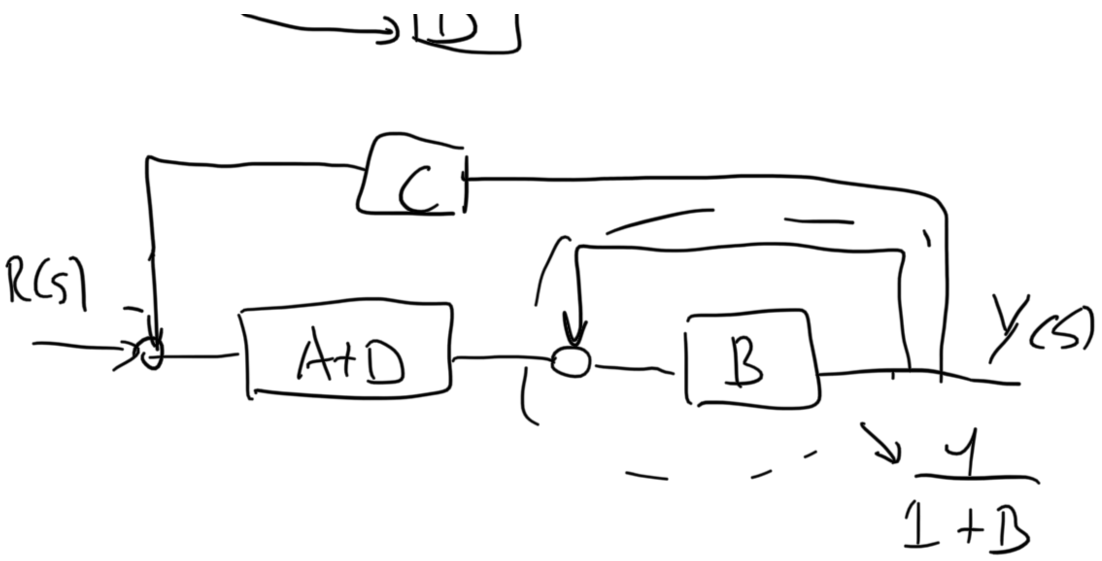
$$\lim_{s \to 0} \cancel{s} \, \frac{6}{\cancel{s^2}} \, \frac{\cancel{s}}{s+3} = \frac{6}{3} = 2$$

FILA B

ESERCIZIO 1

$A = 2$, $B = \frac{1}{s+4}$, $C = 5$, $D = \frac{1}{s}$





$$G(s) = \frac{(A+D)B}{(A+D)BC + 1 + B}$$

sostituendo

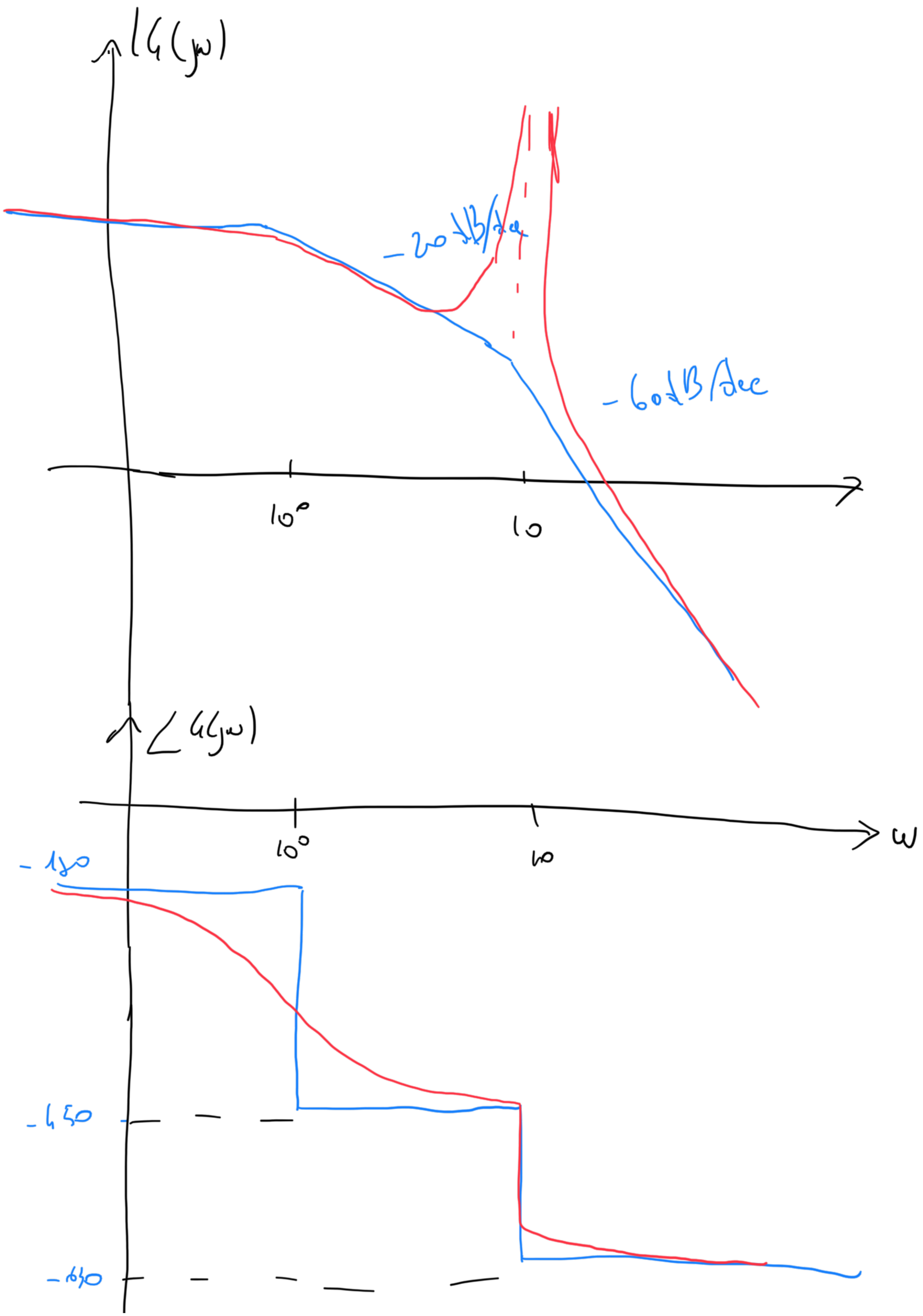
$$G(s) = \frac{\alpha s + 1}{s^2 + (5+5\alpha)s + 5}$$

PER CARTESIO STABILE SE

$$5 + 5\alpha > 0 \longrightarrow \alpha > -1$$

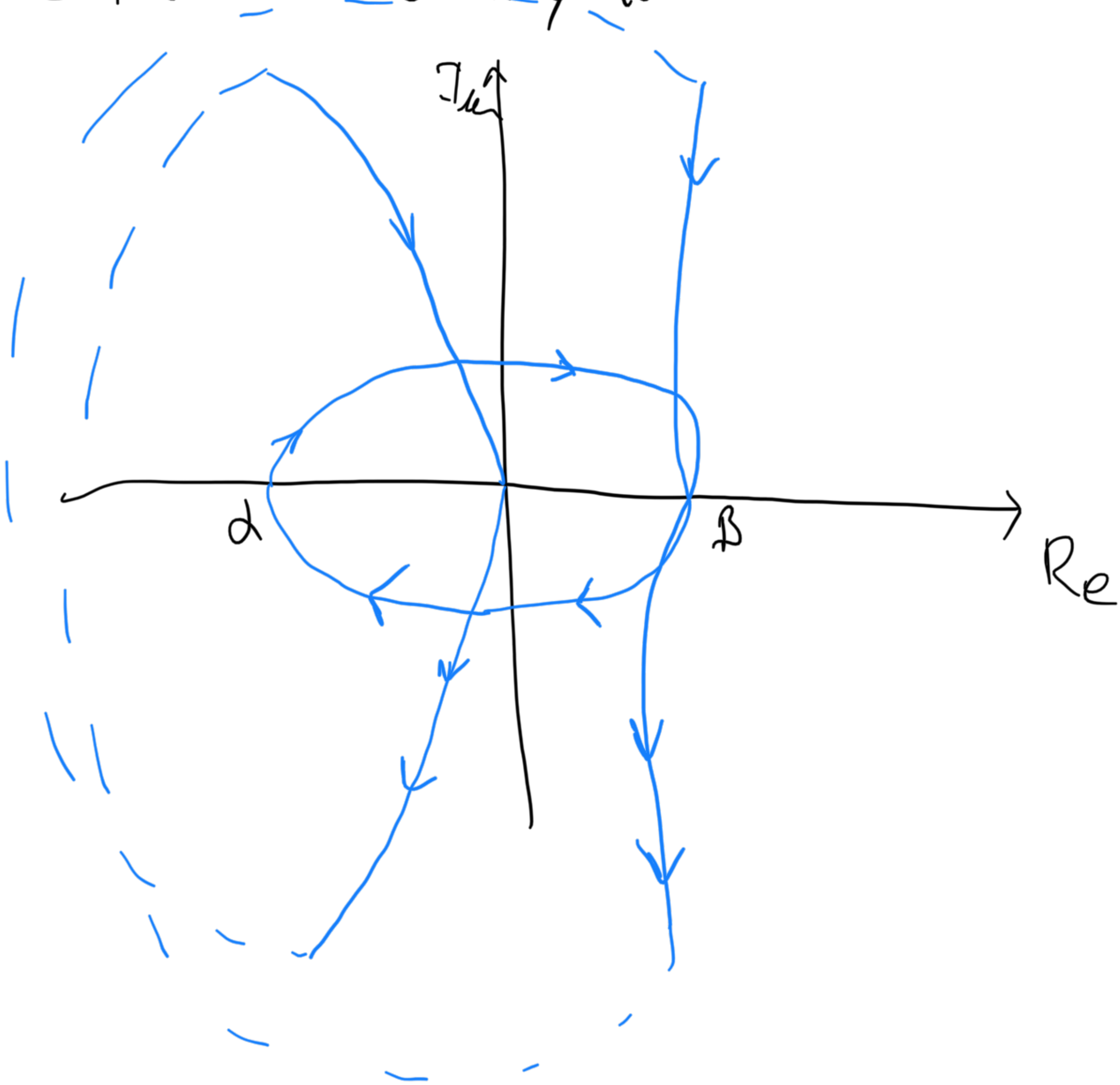
ESERCIZIO 2

$$G(s) = \frac{100(s-1)}{(s+1)^2(s^2+100)} = -\frac{(1-s)}{(1+s)^2\left(1+\frac{s^2}{100}\right)}$$



|G(jw)|
-20dB/dec
-60dB/dec
10⁰
10
∠G(jw)
ω
-180
-450
-630

DIAGRAMMA DI NYQUIST



$k > 0$

$k > 1/2$ 3 poli DX

$k < 1/2$ 2 poli DX

$k < 0$

$k < -1/\beta$ 2 PD

$k > -1/\beta$ STABILE

ESERCIZIO 3

$$G(s) = \frac{(s^2+25)}{s(s+3)^2(s+1)(4-s)}$$

FORMA POLI/ZERI

$$G(s) = -\frac{(s+5j)(s-5j)}{s(s+3)^2(s+1)(s-4)}$$

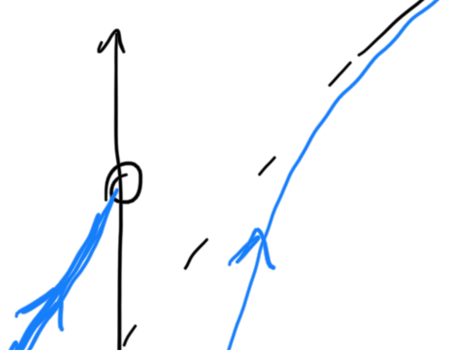
$K<0$

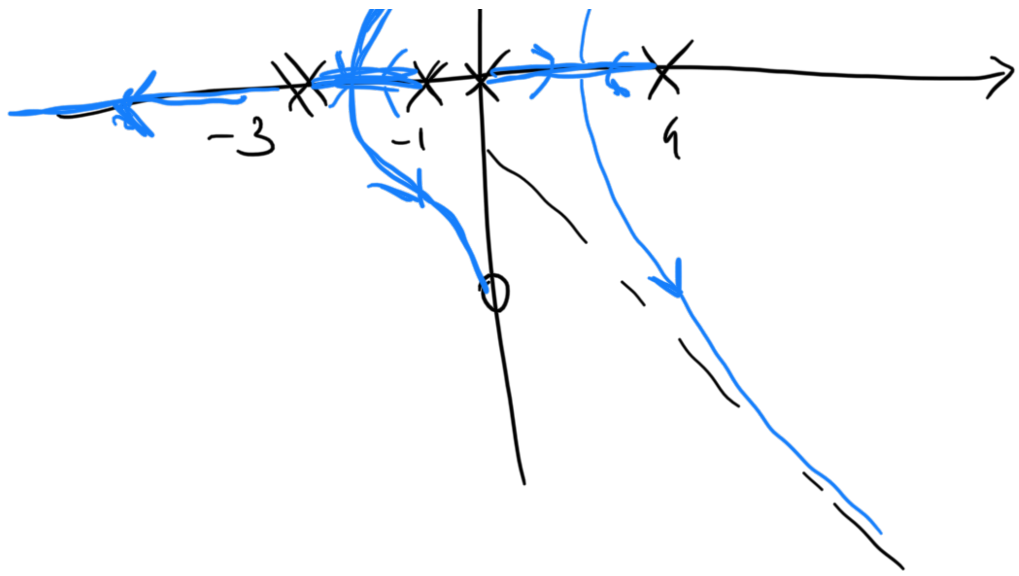
$$s_0 = \frac{-6+1+4}{3} = -1$$

$$\varphi_0 \begin{cases} \pi/3, \ \pi, \ \frac{5}{3}\pi \\ \\ 0, \ \frac{2}{3}\pi, \ \frac{4}{3}\pi \end{cases}$$

$u = 0, 1, 2$

LUOGO DIRETTO

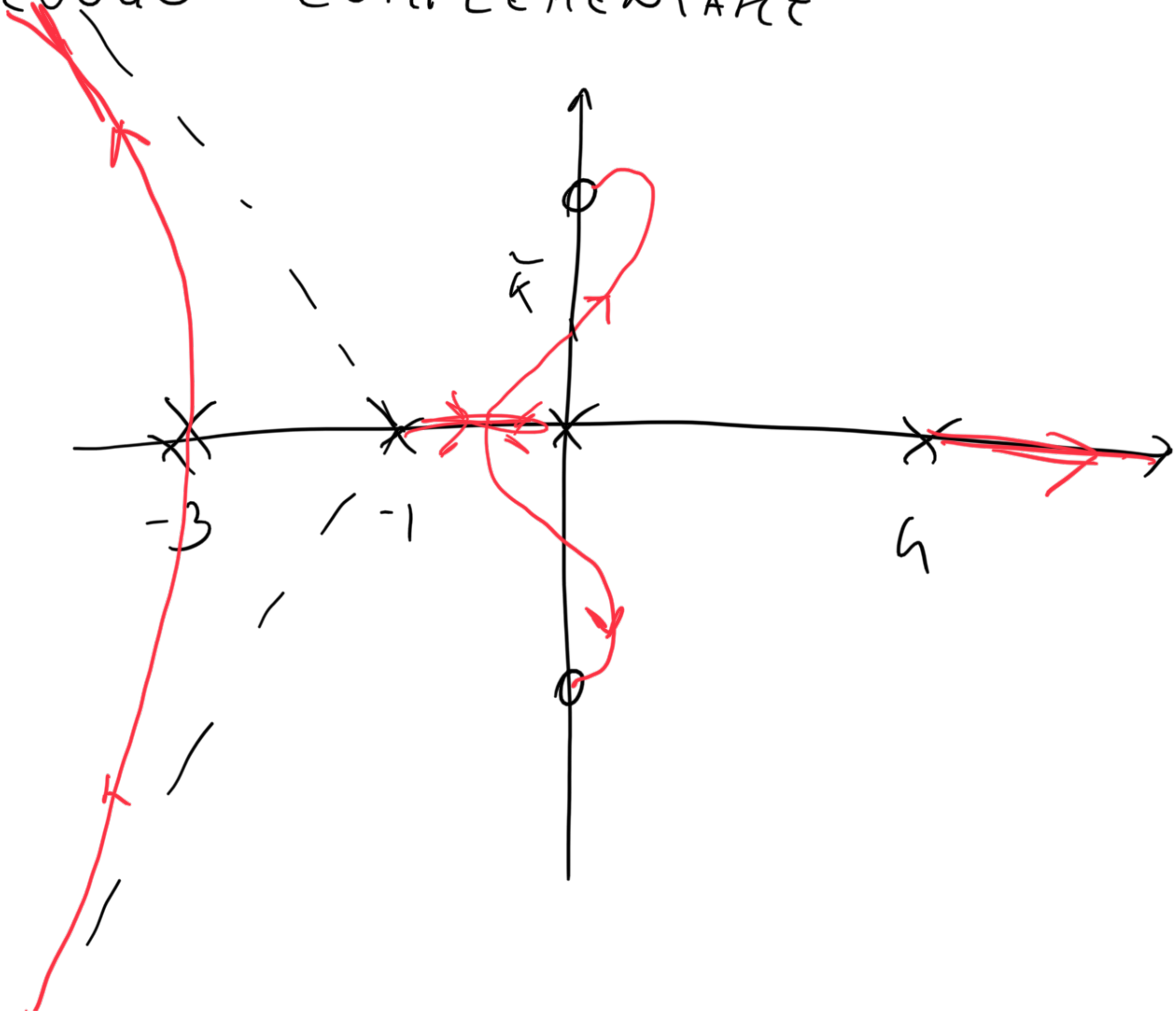




Dalla condizione di fase per $(s-5j)$

$\angle\, s-5j \simeq -112° \rightarrow$ III quadrante

LUOGO COMPLEMENTARE



Dalla condizione di fase

$\angle\, 5 - j5 = 68°$

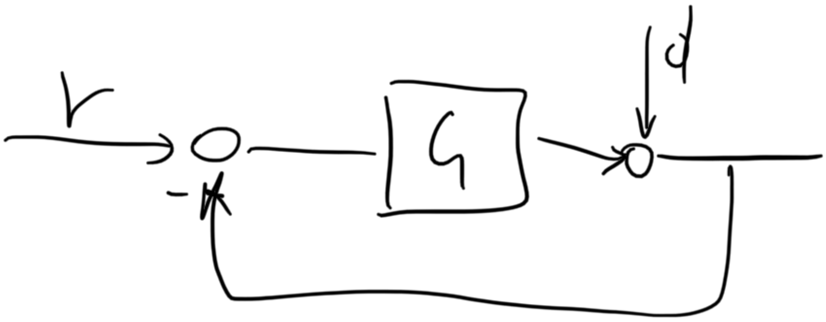
STABILITA'

$K > 0$ (LC) $\quad$ $K < 0$ (LD)

$K > \tilde{K} \rightarrow 3PD$ $\quad$ $2PD$

$K < \tilde{K} \rightarrow 1PD$

ESERCIZIO 4



$G(s)$ sistema tipo 0

$$T(s) = \frac{1}{1+G(s)} = \frac{D(s)}{\varphi(s)}$$

$Y(s) = D(s)T(s)$

$$\ldots = \frac{9}{s} \cdot \frac{1}{s+2}$$

Applico th. valore finale

$$\lim_{s \to 0} \cancel{s} \frac{9}{\cancel{s}} \cdot \frac{1}{3} = 3$$